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Multivariate shift testing for hydrological variables, review, comparison and application

F. Chebana [a,*], M.-A. Ben Aissia [a], T.B.M.J. Ouarda [a,b]

[a] Centre Eau Terre Environnement (ETE), Institut national de la recherche scientifique (INRS), 490 de la Couronne, Québec, QC G1K9A9, Canada
[b] Institute Center for Water and Environment (iWATER), Masdar Institute of Science and Technology, PO Box 54224, Abu Dhabi, United Arab Emirates

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ABSTRACT

Hydrological frequency analysis (HFA) is commonly used for the assessment of the risk associated to hydrological events. HFA is generally based on the assumptions of homogeneity, independence and stationarity of the hydrological data. Hydrological events are often described through a number of dependent characteristics, such as peak, volume and duration for floods. Unfortunately, in this multivariate setting, the verification of the above assumptions is often neglected. When a shift occurs in a data series, it can affect the stationarity and the homogeneity of the data. The objective of this paper is to study tests for shift detection in multivariate hydrological data. The considered shift tests are mainly based on the notion of depth function, except for one test that is considered for comparison purposes. A simulation study is performed to evaluate and compare the power of all these tests with hydrological constraints. A flood analysis application is also carried out to show the practical aspects of the considered tests. The power of the considered tests is influenced by a number of factors, including the sample size, the shift amplitude, the magnitude of the series and the location of the shift in the series.

1. Introduction

In general, in order to perform the statistical analysis of hydrological data a number of fundamental assumptions are required. More precisely, preliminary testing for stationarity, homogeneity and independence is a necessary step in any hydrologic frequency analysis (HFA) study (e.g. Rao and Hamed (2000)). One or more of these assumptions can fail because of a number of reasons. For instance, the assumption of stationarity may not be verified because of a regime shift that can be due to an abrupt change in the watershed characteristics caused by natural or anthropogenic actions on the physical environment, such as deforestation or the construction of a hydraulic structure (e.g. Bobée and Ashkar (1991), Burn and Hag Elnur (2002), Ouarda and El-Adlouni (2011)). Because of the growing evidence concerning climate change, the common assumption of stationarity of hydrologic phenomena may no longer hold. The presence of shifts in data series is highlighted in several hydrometeorological studies, such as floods (Seidou and Ouarda, 2007), precipitation (Beaulieu et al., 2008, 2010; Ouarda et al., 2014; Chen et al., 2016), low-flows (Ehsanzadeh et al., 2011), wind speed (Naizghi and Ouarda, 2016), and temperature data (Jandhyala et al., 2014).

The analysis of multivariate events is of particular interest in several applied fields, including hydrology. Indeed, complex hydrological events, such as floods, droughts and storms are multivariate events characterized by a number of correlated variables. For instance, volume (V), peak (Q) and duration (D) describe floods (Ouarda et al., 2000; Shiau, 2003; Yue et al., 1999). The use of univariate HFA can lead to inaccurate estimation of the risk associated to a given event. Recently, several studies adopted the multivariate framework to treat extreme hydrological events, see e.g. Chebana (2013) for a summary and recent references.

HFA is composed of four main steps: (i) descriptive and explanatory analysis, (ii) verification of the basic assumptions including stationarity, homogeneity and independence, (iii) modeling and estimation, and (iv) risk evaluation and analysis. In the univariate setting, these steps are extensively treated (e.g. Rao and Hamed (2000)). In the multivariate context, the first two steps (i and ii) attracted considerably less attention than the two others. For an overview of step (i) in the multivariate framework, the reader is referred to Chebana and Ouarda (2011). Checking the basic assumptions (step ii) is generally ignored in the hydrological literature in the multivariate setting. For instance, it is not treated

* Corresponding author.
E-mail address: fateh.chebana@ete.inrs.ca (F. Chebana).

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in Kao and Govindaraju (2007), Song and Singh (2009) and Vandenberghe et al. (2010). This step has a significant impact on steps (iii) and (iv). Therefore, ignoring step (ii) may lead to inaccurate models and hence to wrong results and inappropriate decisions regarding resource management and infrastructure design. In order to avoid the loss of human lives and property associated with design event underestimation, or the increase in construction cost associated with overestimation, it is necessary to treat step ii) for a sound and complete multivariate HFA.

Non-stationarity is a very wide notion and includes in particular the presence of one or several shifts in the data. Recently, Chebana et al. (2013) provided a review and application of multivariate non-parametric tests for monotonic trends and presented approaches that can be considered as a preliminary step in a complete multivariate HFA. Chebana et al. (2013) indicated that, for multivariate hydrological data, various types of non-stationarities can be found for which appropriate tests should by reviewed, compared and applied.

The available literature on shift detection in the hydrological context is focused on the univariate setting. Nevertheless, statistical literature exists for the general multivariate setting. Hence, existing comparisons and evaluations of the proposed tests are based on scenarios and hypotheses that are not adapted to the hydrological context (e.g. sample size, scale, and distributions). In addition, these comparative studies are not exhaustive and are often not based on quantifiable performance criteria. Consequently, there is a need for comparative studies that consider all available tests and are representative of hydrological reality, scale and constraints.

Several multivariate shift tests are based on the concept of depth function. The latter is a statistical notion to measure the *depth* (or its opposite, the *outlyingness*) of a given point with respect to a multivariate data cloud or its underlying distribution. Depth functions were developed in the seventies and have been receiving increasing interest (e.g. Tukey (1975), Liu (1990), Zuo and Serfling (2000), Mizera and Müller (2004), Zuo and Cui (2005), Lin and Chen (2006), Liu and Singh (2006), Chebana and Ouarda (2011), Singh and Bárdossy (2012), Lee et al. (2014), Wazneh et al. (2013, 2015)). Depth functions provide a scale-standardized measure of the position of any data point relative to the center of the distribution due to its affine-invariant property (Li and Liu, 2004). For the location shift, this property allows us to view the depth-based test statistics as scale-standardized measures. Therefore, depth-based tests can be performed without the difficulty of estimating the variance of the null sampling distributions. Instead, the decision rule is derived by obtaining p-values using the idea of permutation.

The objectives of the present paper are: (1) to show the importance of the testing step in a multivariate HFA, in particular shift testing, (2) to review shift tests that are available in the statistical literature and which are applicable to hydrological variables within the multivariate HFA context, and (3) to perform an overall evaluation and comparison of these tests under hydrological constraints (such as short sample size, specific distributions).

This paper is organized as follows. Section 2 introduces the definitions and notations related to the shift concept. The considered tests are described in Section 3. The simulation study to evaluate the performance of these tests is presented in Section 4. Section 5 illustrates an application of the reviewed tests on hydrological data. The conclusions of the study and a number of perspectives are reported in Section 6.

2. Shift concept

A shift can be defined by the date at which at least one feature of a statistical model (e.g., location, scale, intercept and trend) undergoes an abrupt change (Seidou et al., 2007). A large number of techniques can be found in the literature to identify the date of a potential shift and to check its significance. Most of the methodologies use statistical hypothesis testing to detect shifts in the slope or intercept of linear regression models (Easterling and Peterson, 1995; Vincent, 1998; Lund and Reeves, 2002). For instance, Solow (1987), Easterling and Peterson (1995), Vincent (1998), Lund and Reeves (2002), and Wang (2003) used the Fisher test to compare a model with and without a shift. The Student and Wilcoxon tests can also be applied sequentially to detect shifts in data series (Beaulieu et al., 2007, 2008).

Note that not all shift approaches are based on hypothesis testing. For instance, Wong et al. (2006) used the grey relational method (Moore, 1979; Deng, 1989) for single shift detection in stream flow data series. In some rare cases, curve fitting methods were used (e.g. Sagarin and Micheli (2001), Bowman et al. (2006)). Extensive reviews of shift detection and correction methodologies in hydrology and climate sciences can be found in Peterson et al. (1998) and Beaulieu et al. (2009).

To define a shift, let $(x_i)_{i=1,\ldots,n}$ be a given d-variate dataset and $1 < s < n$ be a possible shift. If such s exists, the series is divided into two subsamples with sizes s and $m = n\text{-}s$ such that:

$$\begin{aligned}(y_1,\ldots,y_s) &= (x_1,\ldots,x_s)\\ (z_1,\ldots,z_m) &= (x_{s+1},\ldots,x_n)\end{aligned} \tag{1}$$

Denote by G_1 and G_2 respectively the cumulative distribution functions of these two subsamples. The two distributions G_1 and G_2 have the same form, except for the location, i.e. $G_1(x) = G_2(x + \boldsymbol{\delta})$ for all $x \in R^d$ where $\boldsymbol{\delta} \in R^d$ is a constant vector. Consequently, when testing the presence of a shift at a position s of the series $(x_i)_{i=1,\ldots,n}$, the null and alternative hypotheses are respectively:

$$H_0 : \boldsymbol{\delta} = 0 \text{ i.e. there is no location shift} \tag{2}$$

$$H_1 : \boldsymbol{\delta} \neq 0 \text{ i.e. there are two different subsamples at least in one component of } \boldsymbol{\delta}. \tag{3}$$

3. The considered tests

In the present paper, several tests to detect a shift in the location of multivariate series are considered. Except for the C-test, all the presented tests are based on depth functions. The C-tests is considered for comparison purposes. More details are given below regarding p-value evaluation. Table 1 presents a summary of the tests considered in this study.

3.1. Depth functions

The absence of a natural order for multivariate data led to the introduction of depth functions (Tukey, 1975). They are developed and used in a number of research fields, e.g. in statistics by Mizera and Müller (2004), in industrial quality control by Liu and Singh (1993) and in water sciences by Chebana and Ouarda (2008). A detailed description and review of depth functions can be found in Zuo and Serfling (2000). In the following we present a very brief overview of the main concepts. For a given cumulative distribution function F on $\mathfrak{R}^d (d \geqslant 1)$, a depth function can be defined. It is any non-negative bounded function which possesses a number of suitable properties, i.e. *Affine invariance, Maximality at center, Monotonicity relative to the deepest point, Vanishing at infinity*.

A number of depth functions have been developed and studied (Zuo and Serfling, 2000). In the following, we present some of the key ones which are considered in this study: