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The effect of intra-wellbore head losses in a vertical well

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ABSTRACT

Flow to a partially penetrating vertical well is made more complex by intra-wellbore losses. These are caused not only by the frictional effect, but also by the kinematic effect, which consists of the accelerational and fluid inflow effects inside a wellbore. Existing models of flow to a partially penetrating vertical well assume either a uniform-flux boundary condition (UFBC) or a uniform-head boundary condition (UHBC) for treating the flow into the wellbore. Neither approach considers intra-wellbore losses. In this study a new general solution, named the mixed-type boundary condition (MTBC) solution, is introduced to include intra-wellbore losses. It is developed from the existing solutions using a hybrid analytical-numerical method. The MTBC solution is capable of modeling various types of aquifer tests (constant-head tests, constant-rate tests, and slug tests) for partially or fully penetrating vertical wells in confined aquifers. Results show that intra-wellbore losses (both frictional and kinematic) can be significant in the early pumping stage. At later pumping times the UHBC solution is adequate because the difference between the MTBC and UHBC solutions becomes negligible.

1. Introduction

When modeling flow into a well, an important issue is how to handle the vertical screened well face. Presently, there are two ways to deal with it: a uniform-flux boundary condition (UFBC) or a uniform-head boundary condition (UHBC). UFBC assumes that the flux distribution along the screened well face is uniform, and has the advantage of relatively simple implementation in the analysis. The results of UFBC show that the head distribution varies along the screened well face and the lowest head is at the midpoint of the screen, which implies that water is removed from the midpoint of the screen. UHBC assumes that the hydraulic head is uniformly distributed along the screened well face, which implies that the wellbore hydraulic conductivity is infinite. This seems more reasonable than UFBC, since the wellbore hydraulic conductivity is much larger than the aquifer vertical hydraulic conductivity. UHBC is also named non-uniform flux boundary condition (Perina and Lee, 2006) or uniform drawdown boundary condition (Hemker, 1999). It has been widely used to describe the flow into a partially penetrating well, e.g. Hemker (1999), Cassiani et al. (1999), Perina and Lee (2006), Chang and Chen (2003).

However, neither UFBC nor UHBC specifically consider the physical processes of head losses in the wellbore, which will be taken into account in this study. We name the approach considering the intra-wellbore losses along the screened well face as the mixed-type boundary condition (MTBC). It is different from the definition of MTBC in some previous studies (Chang and Chen, 2003; Mathias and Wen, 2015), in which UHBC is sometimes used to refer to MTBC.

The influence of the intra-wellbore head losses on the aquifer test has been observed in many previous studies. It creates complex flow patterns around the well. For instance, in respect to the single-well pumping test in which the pumping well is also used as a monitoring well to measure the water level, Chen and Jiao (1999) developed a model to describe the intra-wellbore head losses from the frictional effect along the well screen using the Darcy-Weisbach formula (Munson et al., 1990) for pipe flow. Hu et al. (2011) investigated the intra-wellbore head losses in the well using a similar approach.

Due to the nature of radial inflow into a pumping vertical well through a screen, the mechanics of flow along the well screen are somewhat different from that of the conventional pipe-flow. In addition to the frictional effect, head losses along the wellbore might be further altered by the kinematic effect. This includes both the change from horizontal to vertical flow and the acceleration due to the increasing flow velocity towards the point of intake

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inside the vertical well. There is evidence that both the frictional effect and the kinematic effect are important factors to consider for a vertical well (Dikken, 1990; Tarshish, 1992, 1993). Tarshish (1992, 1993) proposed a model for steady-state flow into a well, by considering the intra-wellbore head losses from both the frictional effect and the accelerational effect. In addition, numerous studies on the intra-wellbore head losses have been conducted in the petroleum engineering, such as Dikken (1990), Ozkan et al. (1995), Sarica et al. (1994), and Ozkan et al. (1999), who used MTBC to evaluate the well production performance.

Among numerous studies on aquifer tests, a great deal of effort has been devoted to study a partially penetrating vertical well because of its complex three-dimensional flow nature near to the well, and its common application in the field, especially when aquifer thickness is large (Driscoll, 1986; Kruseman and de Ridder, 2000; Yeh and Chang, 2013; Wang et al., 2015). Butler and Zhan (2004) presented a semi-analytical solution of groundwater flow in response to a slug or pumping test in a highly permeable aquifer. Perina and Lee (2006) used Laplace and finite Fourier-cosine transforms to develop a general well function for pumping tests in an unconfined or leaky aquifer considering the effects of a finite-thickness skin and well partial penetration. This general well function could be used to describe the transient hydraulic head response for constant-head tests, constant-rate tests, and slug tests. Yang and Yeh (2012) presented similar solutions for the case of a confined aquifer.

In this study, the intra-wellbore head losses will be investigated for flow into a partially penetrating vertical pumping well, considering both the kinematic and frictional effects inside the wellbore in a confined aquifer. This study advances the present knowledge of pumping test models by developing new MTBC solutions based on the UFBC solutions. The new solutions are not only capable of interpreting the data of the transient hydraulic head response for constant-head tests and constant-rate tests, but can also be applied to well design, pump selection and pump placement in a confined aquifer.

2. UFBC and UHBC solutions of flow into a partially penetrating vertical well

2.1. Well boundary models

With respect to the treatment of the well screen of a partially penetrating vertical well in aquifer tests, three types of boundary conditions are relevant: UFBC, UHBC and MTBC. UFBC and UHBC can be respectively described by Eqs. (1) and (2) as follows

$$\left.\frac{\partial H}{\partial r}\right|_{r=r_w} = F_1(t), \quad \text{when} \quad z_{bot} \leqslant z \leqslant z_{top}, \tag{1}$$

$$H|_{r=r_w} = F_2(t), \quad \text{when} \quad z_{bot} \leqslant z \leqslant z_{top}, \tag{2}$$

where H is the hydraulic head inside the wellbore [L]; $F_1(t)$ and $F_2(t)$ are functions of time t [T]; r is radial distance [L]; r_w is the radius of the well screen [L]; z_{top} and z_{bot} are the vertical coordinates [L] of the top and bottom of the well screen, respectively.

Applying a finite-difference scheme in which the well screen is divided into N equal segments with segment 1 at the top of the screen and segment N at the bottom (Fig. 1), Eqs. (1) and (2) become:

$$q_{i+1} = q_i \quad \text{for} \quad \text{UFBC}, \; i = 1, 2, \ldots, N-1, \tag{3}$$

$$H_{i+1} = H_i \quad \text{for} \quad \text{UHBC}, \; i = 1, 2, \ldots, N-1, \tag{4}$$

where q_i and H_i respectively represent the mean flow rate per unit length from the aquifer to the well [L^2T^{-1}] and the mean hydraulic head [L] at the well screen over the well segment i. If k_{well} is the hydraulic conductivity of the wellbore [LT^{-1}], one can conclude from Eqs. (1) and (2) that k_{well} approaches infinity for UHBC, while for UFBC k_{well} is equal to the vertical hydraulic conductivity of the aquifer, k_z [LT^{-1}]. Note that UFBC and UHBC are identical for a fully penetrating well because there is no vertical flow in the aquifer and the flow within the well is not considered. The well hydraulic conductivity of the MTBC model is higher than k_z of UFBC and lower than infinity of UHBC, and MTBC depends also on the pump location inside the well. Thus MTBC is never identical to UFBC or UHBC.

Due to the complexity of the UHBC and MTBC models, analytical solutions might not be available. Alternatively, the UHBC and MTBC solutions can be obtained by a hybrid analytical-numerical method, in which the analytical solution is based on the UFBC solutions. In the following, we firstly introduce the UFBC solution.

2.2. The UFBC solutions

A general well function of UFBC in a confined aquifer was derived by Yang and Yeh (2012) via the Laplace and finite Fourier transforms considering the effect of well partial penetration. Assumptions include an aquifer of infinite radial extent with uniform thickness. This UFBC solution is capable of solving the problems of different types of aquifer tests (like constant-head tests, constant-rate tests, or slug tests) for confined aquifers. A cylindrical coordinate system has been adopted with the r-axis horizontal, and the z-axis vertically upward. The origin of the coordinate system is at the intercept point of the center of the well and the aquifer bottom. The UFBC solution of Yang and Yeh (2012) in Laplace domain is:

$$\begin{aligned}\bar{d}(r,z,p) = {} & \frac{(\bar{Q} + \pi r_c^2 \overline{d_w})(z_{top} - z_{bot}) K_0(r\sqrt{pS_s/k_r})}{\pi r_c^2 bpK_0(r_w\sqrt{pS_s/k_r}) + 2\pi r_w k_r (z_{top} - z_{bot})\sqrt{pS_s/k_r} K_1(r_w\sqrt{pS_s/k_r})} \\ & + \sum_{n=1}^{\infty} \frac{2(\bar{Q} + \pi r_c^2 \overline{d_w}) \frac{\sin(z_{top} n\pi/b) - \sin(z_{bot} n\pi/b)}{n\pi}}{\pi r_c^2 pK_0(\xi_n r_w) + 2\pi r_w k_r (z_{top} - z_{bot}) \xi_n K_1(\xi_n r_w)} K_0(\xi_n r) \\ & \times \cos(zn\pi/b)\end{aligned} \tag{5}$$

where the over bar implies the variables in Laplace domain; p is the Laplace transform variable; b is the saturated thickness [L]; d and d_w are the drawdowns [L] in the aquifer and the well, respectively; Q is the well pumping rate [LT^{-3}]; k_r is the horizontal hydraulic conductivity [LT^{-1}]; S_s is the specific storage [L^{-1}]; r_c is the radius of the well casing [L]; K_0 is the modified Bessel functions of the first kind and zeroth order; K_1 is the modified Bessel functions of the first kind and first order; z is the vertical coordinate (upward positive);$\xi_n = \sqrt{\frac{k_z}{k_r}\left(\frac{n\pi}{b}\right)^2 + \frac{S_s p}{k_r}}$

Eq. (5) for constant-head tests becomes:

$$\begin{aligned}\bar{d}(r,z,p) = \bar{Q}\Bigg[& \frac{1}{2\pi r_w k_r b} \frac{K_0(r\sqrt{pS_s/k_r})}{\sqrt{pS_s/k_r} K_1(r_w\sqrt{pS_s/k_r})} \\ & + \sum_{n=1}^{\infty} \frac{\sin(z_{top} n\pi/b) - \sin(z_{bot} n\pi/b)}{n\pi} \frac{K_0(\xi_n r)}{\xi_n K_1(\xi_n r_w)} \frac{\cos(zn\pi/b)}{\pi r_w k_r (z_{top} - z_{bot})}\Bigg].\end{aligned} \tag{6}$$

As for constant-rate tests, Eq. (5) becomes:

$$\begin{aligned}\bar{d}(r,z,p) = \frac{Q}{p}\Bigg[& \frac{1}{2\pi r_w k_r b} \frac{K_0(r\sqrt{pS_s/k_r})}{\sqrt{pS_s/k_r} K_1(r_w\sqrt{pS_s/k_r})} \\ & + \sum_{n=1}^{\infty} \frac{\sin(z_{top} n\pi/b) - \sin(z_{bot} n\pi/b)}{n\pi} \frac{K_0(\xi_n r)}{\xi_n K_1(\xi_n r_w)} \frac{\cos(zn\pi/b)}{\pi r_w k_r (z_{top} - z_{bot})}\Bigg].\end{aligned} \tag{7}$$

Eqs. (5)–(7) are the same as Eq. (17), Eq. (18), and Eq. (21) of Yang and Yeh (2012), respectively. Note that Eq. (7) is identical to Eq. (6) except that Q/p replaces $\overline{Q}$ because Q is constant with time.