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Research papers

Hydraulic conductivity of stratified unsaturated soils: Effects of random variability and layering

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ABSTRACT

For simulating flow in heterogeneous porous media it is computationally more efficient to define an equivalent effective (i.e., upscaled) medium rather than considering detailed spatial heterogeneities. In this paper, the effective unsaturated hydraulic conductivity (K) of soils exhibiting random variability, layering, or both is calculated based on numerical simulations of steady-state evaporation from a shallow water table. It is demonstrated that the effective K of randomly-varied coarse-textured soils generally falls between the harmonic and geometric means of the unsaturated hydraulic conductivities of the constituting soils. Layering and random variability when occurring concurrently magnify each other's effects on effective K. As a result, the higher the degree of heterogeneity, the lower the effective K. Therefore, neglecting either random spatial variability or layering in numerical simulations can lead to significant overestimation of water flow in soils.

1. Introduction

Sustainable management of subsurface water resources requires in-depth knowledge of vadose zone flow processes that are mainly governed by soil hydraulic properties (Vereecken et al., 2007). However, the commonly observed high spatial variability (e.g., random variability, layering) of soil hydraulic properties makes this a challenging task. Therefore, the characterization of spatial variability effects on soil hydraulic properties has been of great interest for decades (Miller and Miller, 1956; Willis, 1960; Warrick et al., 1985; Sharma and Luxmoore, 1979; Warrick and Yeh, 1990; Tuli et al., 2001; Khaleel et al., 2002; Zhu and Mohanty, 2002; Lu and Zhang, 2004; Assouline and Or, 2006; Schlüter et al., 2012; Sadeghi et al., 2012a; Deng and Zhu, 2015).

To improve the computational efficiency of numerical simulations and avoid potential challenges associated with detailed characterization of highly heterogeneous porous media, various upscaling approaches have been developed to estimate "effective" hydraulic properties of a hypothetically homogenous medium that is equivalent to the heterogeneous medium. Because the unsaturated hydraulic conductivity (K) as a function of pressure head (h) exhibits higher variability in complex heterogeneous media than the soil water characteristic, most of the proposed upscaling methods have been focused on K (e.g., King, 1989; Kitanidis, 1990; Saucier, 1992; Renard and de Marsily, 1997; Neuweiler and Eichel, 2006; Neuweiler and Vogel, 2007; Samouelian et al., 2007; Hunt and Idriss, 2009).

Mualem (1984) studied the anisotropy of unsaturated layered soils considering the well-known averaging law that holds for saturated conditions (Freeze and Cherry, 1979). He stated that effective K in a layered system equals to the arithmetic and harmonic mean of the individual hydraulic conductivities for flow parallel and perpendicular to the layers, respectively. Yeh et al. (1985) confirmed validity of this general law based on a stochastic analysis framework. However, later Yeh and Harvey (1990) challenged these previous findings illustrating that the geometric mean is a better estimate for the effective K than the arithmetic or harmonic means.

Pruess (2004) further tested the applicability of the harmonic mean as effective K. He applied the van Genuchten (1980) $K(h)$

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function to examine 1D (i.e., layering) and 2D heterogeneities and found that the harmonic mean yields reasonable approximations for unsaturated flow in relatively small domains, while the results were less accurate for larger scales. Tang et al. (2008) studied the flow characteristics of layered soils with the contour bar model based on the composite medium approximation (COMA) approach introduced by Pruess (2004). Their work was limited to layered soils with two different materials and by some means confirmed the validity of COMA for steady state conditions.

The conclusions of Pruess (2004) have been also verified by Warrick (2005) and Sadeghi et al. (2014). Based on the steady state Darcy velocity, they studied effective K for several layered soil profiles consisting of repetitive layers with homogeneous sublayers of various thicknesses. They found that the effective K approaches the weighted harmonic mean of the sublayer unsaturated hydraulic conductivities only when the thickness of each main layer is relatively small. Results of Sadeghi et al. (2014) indicated a nonuniform and complex relationship for the effective $K(h)$ function when sublayers exceeded a certain thickness.

Applying the Gardner (1958) $K(h)$ function, Zhu (2008) studied flow in randomly-varied soils to examine the validity of different averaging approaches for effective K. He concluded that for steady state vertical flow with vertical heterogeneity, the equivalent K falls between the harmonic and geometric means; for coarse-textured soils and higher degrees of heterogeneity K is closer to the geometric mean. Deng and Zhu (2015) investigated the anisotropy and effect of domain size and layer composition in two- and three-layer soils. They suggested that a proper quantification of anisotropy is required for larger flow domains, where the application of the harmonic mean would result in significant errors.

In most of the previous studies, including the work cited above, 1D heterogeneous systems were assumed to be either layered systems consisting of two or more homogeneous layers (e.g., Sadeghi et al., 2014), or a uniformly heterogeneous profile consisting of random spatial variability (e.g., Zhu, 2008). To our best knowledge, there is no study that considers both the effects of layering and random variability concurrently. Because natural layered soils also exhibit random variability, we explore effective $K(h)$ functions for such systems in this study, which is an extension of the Sadeghi et al. (2014) approach that only considered layering effects. It is demonstrated that the neglect of random variability can potentially lead to significant errors when calculating the effective $K(h)$ function for highly heterogeneous soils.

2. Theoretical background

Isothermal steady state evaporation during stage II (i.e., the drying/evaporation front is below the soil surface), can be expressed with the Buckingham-Darcy law (Buckingham, 1907) considering the contributions of liquid and vapor flow to the unsaturated hydraulic conductivity:

$$e = K\left(\frac{dh}{dz} - 1\right) = (K_l + K_v)\left(\frac{dh}{dz} - 1\right) \tag{1}$$

where h is the pressure head (the absolute value is considered for convenience), z is the vertical distance from the water table (WT) to the soil surface (i.e., $z = 0$ at the WT and z = WT depth at the soil surface), e is the steady state evaporation rate, K_l and K_v are the liquid and vapor hydraulic conductivities, respectively, and $K = K_l + K_v$.

Solving for z, Eq. (1) yields:

$$z = \int_0^h \frac{K(h)}{K(h) + e} dh \tag{2}$$

Solution of Eq. (2) yields the pressure head distribution, $h(z)$, above the WT (see Fig. 1 in Sadeghi et al., 2014). For a known evaporation rate (e), Eq. (2) can be solved to determine the vertical distance (D_{max}) between the water table and the drying front (DF):

$$D_{max} = \int_0^{h_{max}} \frac{K(h)}{K(h) + e} dh \tag{3}$$

where h_{max} is the pressure head at the DF.

An exact analytical solution to Eq. (3) is provided in Sadeghi et al. (2012b) for a simple power form of the $K(h)$ function. For more complex $K(h)$ functions (e.g., the van Genuchten-Mualem model), there is no exact analytical solution to Eq. (3). Nonetheless, Sadeghi et al. (2014) derived an approximate analytical solution to Eq. (3) for any arbitrary $K(h)$ function yielding:

$$D_{max} = h_e \tag{4}$$

where h_e is the pressure head at which $K = e$. Eq. (4) states that pairs of $e - D_{max}$ values coincide with the unsaturated hydraulic conductivity curve, $K(h)$, which means that the steady-state evaporation rate exhibits a measure for unsaturated hydraulic conductivity at the pressure head equal to D_{max}. In summary, when $h = D_{max}$, $K = e$. The applicability of this approach for coarse-textured media is due to the assumption of a symmetrical shape of $1/[K(h) + e]$ in the derivation steps, which is not accurate for fine-textured media. Sadeghi et al. (2014) indicated that the resulting $K(h)$ curve holds for the entire D_{max} domain, which means that for heterogeneous soils it represents the "effective" unsaturated hydraulic conductivity of the entire heterogeneous profile. Therefore, this method provides a unique opportunity to directly calculate the effective K curve for various arbitrary heterogeneous soil profiles via forward steady-state simulations based on the Buckingham-Darcy law (Buckingham, 1907). This approach is computationally more efficient and much simpler than the conventional inverse solution approach based on Richards' equation (Richards, 1931). In addition, this new method is not restricted to a specific mathematical form of the $K(h)$ function, and therefore more appropriate than the inverse solution for layered soils, where the effective $K(h)$ curve may substantially differ from the conventional unimodal $K(h)$ functions (Sadeghi et al., 2014).

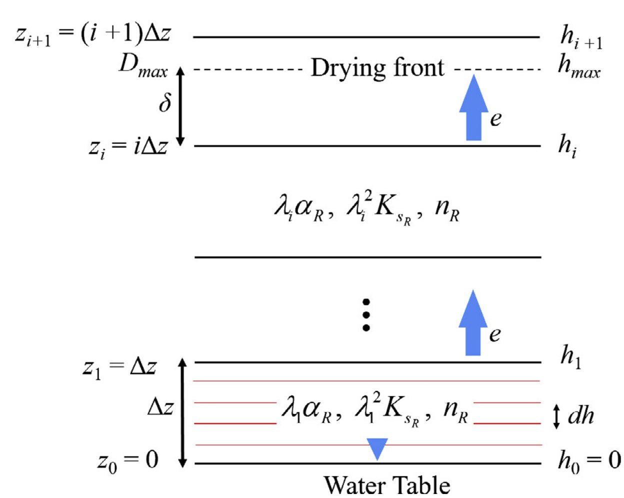

Fig. 1. Sketch depicting the numerical simulation scheme and random variability for a one-layer system. Calculations were performed for i sublayers with thickness Δz, starting from the water table up to the drying front. Hydraulic properties for each layer were determined using a random scaling factor applied to a reference soil. The pressure head at each interface, h_i, was calculated with Eq. (7). Calculations continued until h approached h_{max}. Location of the drying front, D_{max}, was then calculated as $i\Delta z + \delta$, where δ was solved using Eq. (8).