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Technical Note

Note on a modified return period scale for upper-truncated unbounded flood distributions

Earl Bardsley

University of Waikato, Faculty of Science & Engineering, Private Bag 3105, Hamilton, New Zealand

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ABSTRACT

Probability distributions unbounded to the right often give good fits to annual discharge maxima. However, all hydrological processes are in reality constrained by physical upper limits, though not necessarily well defined. A result of this contradiction is that for sufficiently small exceedance probabilities the unbounded distributions anticipate flood magnitudes which are impossibly large. This raises the question of whether displayed return period scales should, as is current practice, have some given number of years, such as 500 years, as the terminating rightmost tick-point. This carries the implication that the scale might be extended indefinitely to the right with a corresponding indefinite increase in flood magnitude. An alternative, suggested here, is to introduce a sufficiently high upper truncation point to the flood distribution and modify the return period scale accordingly. The rightmost tick-mark then becomes infinity, corresponding to the upper truncation point discharge. The truncation point is likely to be set as being above any physical upper bound and the return period scale will change only slightly over all practical return periods of operational interest. The rightmost infinity tick point is therefore proposed, not as an operational measure, but rather to signal in flood plots that the return period scale does not extend indefinitely to the right.

1. Introduction

All hydrological processes have some physical upper bound and we can always specify some flood magnitude that is impossibly large, as opposed to having a vanishingly small probability of occurrence. Rainfall simulation models, for example, may incorporate an upper bound (Costa et al., 2015). Despite this bounded reality, probability distributions unbounded to the right such as the lognormal, Gumbel and gamma distributions often give good fits to annual flood maxima. A display problem then arises in flood plots as to the choice of the rightmost tick-mark on the return period scale. Typically this will be some large round number such as 500 years, but the implication remains that the return period scale could be extended indefinitely to the right.

This is clearly an issue of graph aesthetics rather than operational hydrology. However, there is a scientific implication involved in that there is a need to give visible indication that the return period scale does not in reality extent forever to the right.

One approach which might be attempted here is to revise the return period scale by making the assumption that the annual flood maxima in fact represent random variables from an unknown right-bounded distribution, and then seek nonparametric estimators of that bound. The return period scale would then terminate at an infinity tick point on the right, corresponding to the upper bound. An early nonparametric bound estimator was presented in the statistical literature by Robson and Whitlock (1964), utilising just the largest and second largest values in a sample. Related statistical papers include Cooke (1980), Hall and Wang (1999), Girard et al. (2012), and Alves and Neves (2014). An overview of statistical methodology of bound estimation in the context of regional earthquake magnitudes is given by Kijko and Singh (2011).

However, it would seem unrealistic to employ statistical estimation of upper bounds when annual flood maxima are already well described by a distribution with no upper bound. That is, the largest data values show no evidence of the proximity of a bound because the information content of the data is already summarised by the unbounded distribution parameters. In this situation it is inevitable that statistical upper bound estimates will have such large estimation errors as to have little practical value for identifying a true physical bound. One potential improvement for flood distributions might come from fitting right-bounded distributions where the bound is estimated with the aid of physically-

E-mail address: earl.bardsley@waikato.ac.nz

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based estimates of probable maximum flood magnitudes (Botero and Francés, 2010; Fernandes et al., 2010).

This brief communication proposes a simple alternative approach to the bound issue in return period scales, consistent with recorded data. The method replaces well-fitted unbounded distributions of annual flood maxima with the same distribution but now incorporating a high upper truncation point. The truncation point then serves as the infinity tick-point to terminate the return period scale on the right. There is no implication of specifying a physical upper bound and the truncation point must be selected somewhere beyond the feasible flood range. With this proviso, the truncation point location is not critical through some checking might be required as to how large a feasible flood may be (England et al., 2014). That is, the focus here is on an improved scale representation and not hydrological discovery or parameter estimation methodology. The return period scale over the region of operational interest will remain essentially unchanged provided the truncation point is selected to be sufficiently high. Time scales of operational interest are assumed here to be in the order of a few hundred years because extreme right-truncation would be required to be far beyond a deemed operational return period of, say, 10,000 years.

Inevitably, extending a return period scale to the right will involve greater display of white space on a flood plot and the choice of displaying a right infinity point becomes one of personal preference. However, the introduction of an upper truncation point also has a practical advantage in that extending the return period toward infinity for unbounded distributions no longer implies flood magnitudes extending to infinity.

It would appear that this is the first proposal for this simple right-truncation concept to be used for return period scales. A right-truncation approach is mentioned briefly by Sisson et al. (2006) for the generalised extreme value distribution (GEV). However, the method was unusual in that the recommendation was to use the untruncated GEV if the upper bound was poorly defined, which implies still using the unbounded EV1 or EV2 distributions (Type 1 and Type 2 extreme value distributions respectively).

2. Scale modification

The usual distributional assumption is made that a stationary process generates annual flood maxima, which are taken to be independent random variables from some common distribution. Reference to long return periods here implies exceedance probabilities under the current conditions and not necessarily stationarity for the return period duration.

Define $F(x)$ as a cumulative distribution function unbounded on the right, applied to a set of annual flood maxima. The standard relation giving return period $R_{F(x)}$ is:

$$R_{F(x)} = 1/(1 - F(x)) \quad x < \infty \tag{1}$$

Now introduce a finite upper truncation point β somewhere well beyond the largest data value, which defines the truncated distribution $G(x)$:

$$G(x) = p^{-1}F(x) \quad x \le \beta \tag{2}$$

where $p = F(\beta)$. The new return period relation then becomes:

$$R_{G(x)} = 1/(1 - G(x)) \tag{3}$$

Introducing truncation at β has the effect of producing a longer return period for any given discharge $x \leqslant \beta$. However, this difference will only become evident when x is sufficiently close to β such that the return period ratio $R_{G(x)}/R_{F(x)} = (1-F(x))/(1-G(x))$ is not near 1.0. Given operational return periods up to 500 years and it is desired that the two time scales are not very different up to this point, $p = 0.9999$ is suggested as giving similar return periods with $R_{F(x)} = 500$ years and the corresponding $R_{G(x)} = 526$ years.

3. The EV1 and EV2 cases

The return period rescaling operation is illustrated with respect to the upper-unbounded EV1 and EV2 extreme value distributions. If $F(x)$ is an EV1 (Gumbel) distribution of largest extremes then $G(x)$ is given by:

$$G(x) = p^{-1}\exp(-e^{-(x-\xi)/\theta}) \quad \theta > 0, \quad -\infty < x \leqslant \beta \tag{4}$$

where θ is a scale parameter and ξ is a location parameter.

If $F(x)$ is an EV2 distribution of largest extremes then $G(x)$ is given by:

$$G(x) = p^{-1}\exp[-((x-\omega)/\alpha)^{1/k}] \quad \alpha > 0, \quad k < 0, \quad \omega \leqslant x \leqslant \beta \tag{5}$$

where α is a scale parameter, ω is a location parameter (lower bound), and k is a shape parameter.

For the extreme value distributions it is common practice to plot annual flood maxima on the vertical axis and have on a horizontal axis $-\ln[-\ln(F(x))]$ (referenced here as z), which gives a linear plot when $F(x)$ is an EV1 distribution.

Keeping in mind the intended use of the horizontal z scale to fix the position of the modified return period scale, for both the right-truncated EV1 and right-truncated EV2 distributions the return period can be written:

$$R_{G(x)} = [1 - p^{-1}F(x)]^{-1} = [1 - p^{-1}\exp(-e^{-z})]^{-1} \tag{6}$$

Solving for z in Eq. (6) gives an explicit expression giving specific points on the horizontal z scale corresponding to the return period values of the modified return period scale:

$$z_{R\ G(x)} = -\ln\left\{-\ln\left[p\left(1 - R_{G(x)}^{-1}\right)\right]\right\} \qquad R_{G(x)} > 1 \tag{7}$$

In contrast, the corresponding standard extreme value distribution expression for z with no upper truncation of the distribution is:

$$z_{R\ F(x)} = -\ln\left[-\ln\left(1 - R_{F(x)}^{-1}\right)\right] \quad R_{F(x)} > 1 \tag{8}$$

Eq. (7) can be used to construct the modified return period scale when p has been specified. For example, if $p = 0.9999$ it is evident that the infinite return period tick-point for the upper discharge bound is obtained as corresponding to $z = 9.21$.

4. Examples

Fig. 1 illustrates an example application to a recorded data set which is reasonably well fitted by an EV1 (Gumbel) distribution, with EV1 parameter estimates of $\xi = 4212$ m^3 s^{-1} and $\theta = 1158$ m^3 s^{-1} obtained from fitting a straight line. The 10,000 year return period magnitude from these parameters (14,877 m^3 s^{-1}) was selected as the upper truncation point in this case. The modified return scale as constructed from Eq. (7) is shown as the upper of the two return period scales in Fig. 1, terminating at the rightmost tick-point with infinite return period corresponding to z = 9.21. This corresponds to the upper truncation point discharge of 14,877 m^3 s^{-1}. The lower return period scale is the usual scale from Eq. (1), displayed for the purposes of comparison. It is evident that there is very little difference between the two scales in this case for return periods to 1000 years.

The second example (Fig. 2) illustrates the effect of moving the upper truncation point too close to the largest recorded discharge value. In this case an EV2 distribution gives an approximation to