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Physically sound formula for longitudinal dispersion coefficients of natural rivers

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ABSTRACT

The longitudinal dispersion coefficient (k) is necessary for a plethora of mass transport applications in fluids, but a general formulation for k remains lacking. In this study, we propose a canonical form for k that reflects the physics of dispersion and suits complex flow conditions encountered in natural streams. This general form is much more concise than previous predictors. A predictor for k of natural streams is also obtained using a genetic programming(GP) without pre-specified correlations among field data or a pre-specified form of the predictor. This predictor is physically sound (i.e. exhibits the aforementioned canonical form) and appears to be commensurate to or better than previous estimates of k. A grey model, which measures the proximity of data to a target shape (i.e. the proposed physically sound form), is also used to verify that the canonical form is appropriate. A formulation for k in natural rivers is obtained by utilising a GP. Its form is consistent with the canonical form.

1. Introduction

Understanding the transport of matter (mass, momentum and heat) in solvent (gas and liquid) is necessary in a plethora of applications, such as contamination control, sediment deposition, flow with vegetation, water intake, and thermal discharge (Burns and Meiburg, 2012; Cassol et al., 2009; Chen et al., 2011; Deng et al., 2001; Escobar, 2015; Guerrero and Skaggs, 2010; Jin et al., 2015; Miño et al., 2013). In the ideal case of passive scalars (mass) in still water, mass flux q is given by Fick's law$q = -Ddc/dx$, where c is the scalar concentration, x is the distance along the longitudinal direction and D is the diffusion coefficient caused by Brownian motion that is influenced by fluid temperature and the size of the molecules of scalar c. In a moving fluid, the transport of scalar mass is conventionally explored in a coordinate system moving at the same average velocity as the fluid but without changing the molecular properties of c (i.e. diffusion coefficient = D). However, the effective 'diffusion coefficient', defined as $-q(dc/dx)$ in moving fluids along x, appears to be much larger than D (Abderrezzak et al., 2015; Aris, 1956; Chen et al., 2012; Fischer, 1979; Ng and Zhou, 2012; Taylor, 1953) and is the theme of the present study. This 'virtual diffusion coefficient', which is commonly referred to as the longitudinal dispersion coefficient k, is not associated with molecular motion. It is the result of macroscopic flow properties associated with the average of the advective acceleration in the longitudinal direction and bulk mixing in the lateral direction (Taylor, 1953; Wu and Chen, 2014a, 2014b; Zeng et al., 2015).

Under certain circumstances, k can be derived by solving the advection–diffusion equation in laminar flow within a circular pipe, in turbulent flow within a circular pipe, in laminar flow in an elliptical pipe, in laminar flow within two planes, in laminar flow in an open channel and in turbulent flow in an open channel. However, k in natural streams does not have a complete theoretical predictor because natural streams have many irregular factors, such as dead zones and vegetation (Huai et al., 2012; Lees et al., 2000).

However, because dispersion in natural rivers shows the same mechanics (i.e. k results from the combination of concentration and velocity gradients in the lateral direction), this study begins with dispersion in laminar flow in a circular pipe (the deduction for dispersion in natural rivers can be found in Text s1). We attempt to discover the generality among the analytical formulae and discuss the dispersion in the aforementioned circumstances, which is a relevant topic worthy of exploration. These analytical formulae are the foundation for obtaining the general formula for k because they are all theoretical solutions and share an interesting identity.

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One of the earliest formulations for k put forward by Taylor (1953) was derived for laminar flow in a pipe. Both theoretical and experimental results showed that $k/D \gg 1$ as a result of the mean velocity gradient in the cross section. The theoretical solution for k is obtained by solving the advection–diffusion equation given as

$$D\left(\frac{\partial^2 c}{\partial r^2}+\frac{1}{r}\frac{\partial c}{\partial r}+\frac{\partial^2 c}{\partial x^2}\right)=\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x} \tag{1}$$

The velocity (u) distribution in a laminar pipe can be derived from the Navier–Stokes equation in radial coordinates; it is given as $u = u_0(1 - r^2/a^2)$, where r is the radial distance from the centre line, a is the radius of the pipe and u_0 is the velocity along the centre line (i.e. maximum velocity). Assuming quasi-steady state conditions and noting that the diffusion gradients along r are much larger than their counterparts along x (i.e. boundarylayer approximation) in long pipes, the third term on the left-hand side and the first term on the right-hand side of Eq. (1) can be neglected. By setting $z_1 = r/a$ and $x_1 = x - u_0t/2$, the advection–diffusion equation for the coordinate system moving at a mean speed U ($=u_0/2$) is

$$\left(\frac{\partial^2 c}{\partial z_1^2}+\frac{1}{z_1}\frac{\partial c}{\partial z_1}\right)=\frac{a^2u_0}{D}\left(\frac{1}{2}-z_1^2\right)\frac{\partial c}{\partial x_1} \tag{2}$$

The formula for k in a laminar pipe flow (see Formula (T1.a) in Table 1) is obtained by solving the equation for the boundary conditions $\partial c/\partial z_1 = 0, at: z_1 = 1$ and by letting $\partial c/\partial x_1$ be independent of z_1. However, k in a turbulent circular pipe flow is different from that in a laminar circular pipe case. The differences are due to the entire mean velocity distribution not having a theoretical formula and the mixing coefficient in the lateral direction being different from the diffusion coefficient in laminar flow as a result of turbulence. Taylor (1954) obtained k for a turbulent circular pipe flow through two experimental results. (1) The experiential velocity distribution over a cross section of a pipe was $(u_0 - u)/u_* = f(z_1)$, where u_* is the shear velocity given as $u_* = (\tau_0/\rho)^{1/2} = (gaJ/2)^{1/2}$ for the pipe; here,$\tau_0$is the shear stress on the wall, g is the gravitational acceleration and J is the energy slope ($f(z_1)$ is omitted here) (Taylor, 1954). (2) Reynolds analogy—the transfers of mass, momentum and heat by turbulence can be connected—was used, i.e. $\varepsilon_r = \tau/(\rho\partial u/\partial r) = -q_r/(\partial c/\partial r)$, where ε_r is the transfer coefficient (a turbulent mixing coefficient in the radial direction, which is much larger than D), τ is the shear stress at radius r and q_r is the concentration flux in the radial direction (Taylor, 1954). By substituting diffusion coefficient D with radial mixing coefficient ε_r and using the experimental mean velocity distribution, k for a turbulent circular pipe flow can be obtained. Specifically, k (see Formula (T2.a) in Table 1) comprises two parts, namely, longitudinal advection and longitudinal mixing, the combination of which results in the longitudinal dispersion (Taylor, 1954).

Aris (1956) obtained a formula for k (see Formula (T3. a) in Table 1) for a laminar elliptical pipe flow; in this formula, the ratio of the major axis (a_1) to the minor axis (b_1) isr_s. The formula recovers the one obtained by Taylor (1953) (see Formula (T1.a) in Table 1) when $r_s = 1$. Extensive studies have also been conducted on k for the laminar flow between two wide planes, the distance between which is H_t, with consideration of the velocity variations in the direction perpendicular to the plane. Another formula for k

Table 1
Formulae for k.

Authors	Formulae	
Taylor (1953)	$k = a^2U^2/(48D)$;	(T1.a)
	$k = [aU/(48D)]aU$	(T1.b)
Taylor (1954)	$k = 10.05au_* + 0.052au_*$ or $k = 7.14aU\gamma^{0.5}$, where u_* was $(gaJ/2)^{1/2}$ for a pipe,	(T2.a)
	J was the energy slope and γ was resistance coefficient	
	$k = (10.1/C_*)aU$	(T2.b)
Aris (1956)	$k = U^2a_1b_1[(5 + 14r_s^2 + 5r_s^4)/(12(r_s + r_s^3))]/(192D)$	(T3.a)
	or	
	$k = \{Ub_1[(5 + 14r_s^2 + 5r_s^4)/(12(r_s + r_s^3))]/(192D)\}a_1U$	(T3.b)
Dewey and Sullivan (1979)	$k = U^2H_t^2/(210D)$	(T4.a)
	or $k = [UH_t/(210D)]UH_t$	(T4.b)
Chatwin and Sullivan (1982)	$k = 2H^2U^2/(105D)$	(T5.a)
	$k = [2HU/(105D)]HU$	(T5.b)
Elder (1959)	$k = 5.93Hu_*$, where u_* is shear of the channel	(T6.a)
	or $k = (5.93/C_*)HU$	(T6.b)
Chikwendu (1986)	for laminar channel flow:	
	$k = 2H^2U^2/(105D) + D$	(T7.a)
	for turbulent channel flow:	
	$k = 0.4041Hu_*/\kappa^3 + \kappa Hu_*/6$	
	where κ is von Karman constant	(T7.b)
	for laminar pipe flow:	
	$k = a^2U^2/(48D) + D$	(T7.c)
Fischer (1975)	$k = 0.011U^2B^2/Hu_*$	(T8.a)
	$k = (0.011UB/Hu_*)BU$	(T8.b)
Liu (1977)	$k = 0.18(u_*/U)^{1.5}(UB)^2/(Hu_*)$	(T9.a)
	$k = (0.18(u_*/U)^{1.5}(UB)/(Hu_*))BU$	(T9.b)
Bogle (1997)	$k = 0.011U^2B^2/Hu_*/(50 \sim 25)$	(T10.a)
	$k = 0.011UB/Hu_*/(50 \sim 25)BU$	(T10.b)
Seo and Cheong (1998)	$k = 5.92(U/u_*)^{1.43}(B/H)^{0.62}Hu_*$	(T11)
Deng et al. (2001)	$k/(Hu_*) = 0.15/(8\varepsilon_d)(U/u_*)^2(B/H)^{5/3}$ where $\varepsilon_d = 0.145+(1/3520)(U/u_*)(B/H)^{1.38}$	(T12)
Kashefipour and Falconer (2002)	$k = [7.428 + 1.775(B/H)^{0.62}(u_*/U)^{0.572}]HU(U/u_*)$	(T13)
Sahay and Dutta (2009)	$k/Hu_* = 2(B/H)^{0.96}(U/u_*)^{1.25}$	(T14)
Azamathulla and Ghani (2011)	$k/Hu_* = \exp\{\exp[\cos(U/u_*)] + [(U/u_*)^2/(B/H + 3.956)]\} + \sin[BU/(Hu_*)] * BU/Hu_*/\exp[\sin(B/H)]$	(T15)
	$+ U/u_*/1.037 - 10.76 * B/H/(U/u_* - 11.38)$	
Etemad-Shahidi and Taghipour (2012)	$k = 15.49(B/H)^{0.78}(U/u_*)^{0.11}H\,u_*$, if $B/H<=30.6$;$k = 14.12(B/H)^{0.61}(U/u_*)^{0.85}H\,u_*$, if $B/H > 30.6$	(T16)
Li et al. (2013)	$k = 2.828(B/H)^{0.7613}(U/u_*)^{1.4713}H\,u_*$	(T17)
Zeng and Huai (2014)	$k = 5.4(B/H)^{0.7}(U/u_*)^{0.13}HU$	(T18)
Disley et al. (2015)	$k/Hu_* = 3.563F_r^{-0.4117}(B/H)^{0.6776}(U/u_*)^{1.0132}$, where F_r is Froude number, and $F_r = U/(gH)^{0.5}$	(T19)
Sattar and Gharabaghi (2015)	$k/Hu_* = 2.9*4.6^{(Fr)\wedge 0.5}Fr^{-0.5}(B/H)^{0.5-Fr}(U/u_*)^{1+(Fr)\wedge 0.5}$	(T20)
Wang and Huai (2016)	$k = 17.648(B/H)^{0.3619}(U/u_*)^{1.16}H\,u_*$	(T21)