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Technical Note

Prediction of oil contamination distribution in aquifers using self similar solutions

Arieh Pistiner

Unit for Hydrocarbon Pollution Prevention, Ministry of Environmental Protection, 15 a Pal-Yam St., Haifa 33095, Israel

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ABSTRACT

Oil contaminant migration in an aquifer is analyzed by applying some power law relationships between the porous medium parameters and oil saturation. Such an application generates a self-similar model whose solutions are used to analyze the effect of the porous structure and the oil properties on the oil migration in the aquifer. By using hypothetical saturation data, the model was used to find the characteristic length and time scales of the aquifer, and then to predict the temporal saturation distribution of the oil contamination in the aquifer.

1. Introduction

In the process of penetration into an aquifer some quantities of the oil are trapped as discrete ganglia in the porous medium (Payatakes, 1982). Generally, the oil is made immobile in a water-saturated porous medium since the water pressure is unable to overcome the capillary pressure required to mobilize the oil. As a result, oil blobs become trapped in the pores, and once disconnected from each other they can no longer flow (Payatakes, 1982; Deng et al., 2015; Mogensen and Stenby, 1998). Such trapped oil quantities become stagnated and comprise the residual oil saturation in the porous medium.

In experiments of imbibition, the nonwetting phase moves slower than the wetting phase, due to the tendency of the latter to bypass the nonwetting phase. If the characteristic size of some pores is comparatively large and that of other pores comparatively small, then the phenomenon of "snap-off" may take place in which the oil body is subject to rupture followed by the trapping of some oil quantities, i.e., water swelling around the oil until it snaps off in the pore throats and traps the oil in globules (Deng et al., 2015).

In contrast with the above, during the drainage of oil against the water flow direction, the oil phase almost stagnates since it achieves an equilibrium state with the water phase for an asymptotic long time. In accordance with this situation, the purpose of this study is to develop a quantitative model, capable of simulating the equilibrium state of oil with the water phase, as oil drainage against the water flow direction. That is to say, the equations of two-phase oil-water transport in the aquifer are used to calculate the unsteady state distribution of oil by utilizing the symmetry properties of the governing equations (Barenblatt, 1979). This calculation yields several valuable similarity solutions and in particular, the extension of the stagnated zone. This zone can be viewed also as a "pseudo residual" state of the oil phase. On the basis of these solutions, several type curves (Bear, 1988) for various porous medium and oil properties are drawn. The model is calibrated by matching a type curve with field data, taken from an observation well and is further used to predict the position of the oil saturation front.

2. Physico-mathematical model

Consider a uni-directional propagation (along the $\hat{x}$ axis) of oil contamination injected into a water saturated, homogeneous, isotropic and non-expansive aquifer. The oil contamination source (e.g., a leaking underground storage tank) is located at $\hat{x} = -0$ and the oil is continuously drained against the water flow direction (see Fig. 1). The outlet edge of the aquifer (at $\hat{x} = 0$) is a horizontal, semi-permeable geological formation that is impermeable to oil

E-mail address: ariehpistiner@gmail.com

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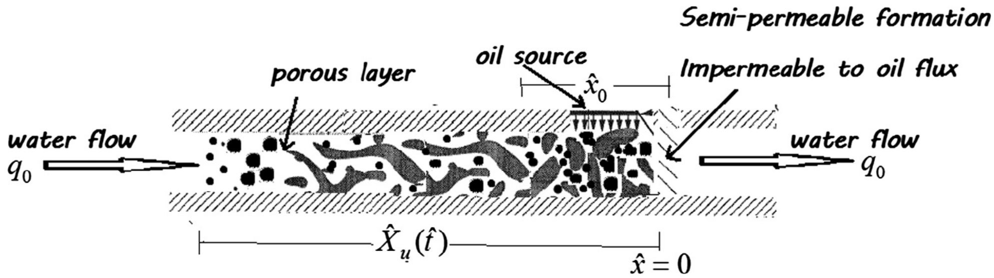

Fig. 1. Schematic representation of the oil saturation profile in the upstream direction of the aquifer.

flux (and permeable to water). The change in stratigraphy of the porous layer, which composes the outlet edge of the aquifer, is moderate and does not occur immediately. However, the length of the semi-permeable layer (e.g., in the $\hat{x}$ direction) is small compared with the dimension of the aquifer. As such, the semi-permeable formation can be viewed as a horizontal line, located in $\hat{x} = 0$, compared with the dimension of the aquifer. In accordance with the above, the oil accumulates within the slab near the barrier, thereby leading to a saturation build-up – capillary end effect (Pistiner and Shapiro, 1993).

Neglecting the gravity forces and combining the Darcy and mass conservation laws for oil and water, we obtain an equation for the oil saturation S (Pistiner et al., 1990):

$$\frac{\partial S}{\partial \hat{t}} + \frac{\partial J}{\partial \hat{x}} = 0, \tag{1a}$$

$$J = q(t)S^n - S^{n-m}\frac{\partial S}{\partial \hat{x}}, \quad (\widehat{X}_u(\hat{t}) \leqslant \hat{x} < 0) \tag{1b}$$

where $\hat{x}$ and $\hat{t}$ are the dimensionless length and time respectively. The length coordinate x and the time coordinate t are related to the characteristic length L and the characteristic time T as follows:

$$x = \hat{x}L, \tag{1c}$$

$$t = \hat{t}T, \tag{1d}$$

and $\widehat{X}_u(\hat{t})$ is the temporal position of the moving saturation front in the upstream direction and $q(\hat{t})$ is the water flux in the porous medium.

The dimensionless parameters m and n, appearing in (1b), were found to dependent on the porous medium and the oil properties. In particular, the parameter n is an increasing function of the oil viscosity, and the coefficient m increases with each decrease of the pore-size index of the porous medium (Pistiner et al., 1990; Pistiner, 2007). From the experimental data on fractional flow rates and permeabilities, it was found that m and n typically vary in the following ranges: $1 < n < 3$; $1.5 < m < 4$.

The distribution of oil saturation is determined by the sign of $n - m$. When $n - m > 0$, the second (diffusion-like) term in the right-hand side of (1b) is negligible for small S. In this case the oil saturation is affected mainly by the convection mechanism. This leads to a situation where the saturation upstream dimensionless front position $\widehat{X}_u(\hat{t}) = X(t)/L$ is located at a finite distance from the oil source, and the evolution of the front is characterized by a free-boundary-type motion (Pistiner et al., 1990). If $n - m \leqslant 0$, the diffusion term in the right-hand side of (1b) may be large when $S \to 0$. In this case the diffusion is the dominant transport mechanism, affecting the saturation profile and the effect of convection is relatively small. Mathematically, since the diffusivity exhibits a singular behavior at $S \to 0$, the saturation profile extends to infinity (i.e. $\widehat{X}_u(\hat{t}) \to -\infty$). In accordance with the above, we will employ the following boundary condition:

$$S = 0, \quad \hat{x} = \widehat{X}_u(\hat{t}), \quad \hat{t} > 0, \tag{2a}$$

$$J = 0, \quad \hat{x} = \widehat{X}_u(\hat{t}), \quad \hat{t} > 0, \tag{2b}$$

imposed upon S and the oil flux J, at the moving oil saturation front.

In principle, a boundary condition for oil saturation should be specified on the oil source, lying in the vicinity of a semi-permeable barrier. However, in a real uni-dimensional model the oil source possesses a finite dimension (for example, a continually leaking underground storage tank) along the $\hat{x}$-axis. As such, no saturation value at the point $\hat{x} = 0$ can be rigorously formulated. However, one may assume that the irreducible water saturation is equal 0.2 and hence, the oil saturation in the aquifer possesses the maximal value (i.e., $S = 0.8$) at a point, which is located in the upstream direction at a distance $\hat{x}_0$ from the origin, where $\hat{x}_0$ is equal to the dimension of the oil source (not necessarily the dimension of an oil tank). This assumption may yield the following boundary condition

$$S(-\hat{x}_0, \hat{t}) = 0.8, \tag{2c}$$

where the yet unknown value of $\hat{x}_0$ will be determined in the course of the solution.

The problem posed by (1a) and (1b), subjected to boundary condition (2a–2c) describes a drain of oil, resulting from a continuously leaking source into the aquifer against the water flow direction, where the leaking source lies in the vicinity of a semi-permeable barrier. The above-mentioned problem starts from a certain initial (at time $\hat{t} = 0$) distribution of the saturation profile $S_0(\hat{x}) = S(\hat{x}, 0)$. The latter profile governs the saturation distribution at an early period (i.e., $\hat{t} > 0$). However, as will be shown later, the long-time saturation profile will prove to be independent of the specific form of $S_0(\hat{x})$. This saturation distribution will be investigated in the next section by the similarity method.

3. The similarity model

We now refer to circumstances in which the saturation profile achieves a certain asymptotic (long-time) form which can be described by a single independent variable ξ (Barenblatt, 1979; Pistiner et al., 1990). To this end, we look for a transformation that reduces the nonlinear PDE (1a), (1b) into an ODE to construct a similarity solution $f(\xi)$ in the form

$$S = f(\xi)\hat{t}^a, \tag{3a}$$

$$\xi = \hat{x}\hat{t}^a, \tag{3b}$$

where a is a negative constant to be determined below. Introducing (3a) and (3b) into Eqs. (1a) and (1b), and assuming that the water flow rate is constant (i.e., $q(\hat{t}) = q_0$), reduce the partial differential