Journal of Hydrology 543 (2016) 625–629

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Technical Note

Probability density functions of the stream flow discharge in linearized diffusion wave models

Ching-Min Chang, Hund-Der Yeh *

Institute of Environmental Engineering, National Chiao Tung University, Hsinchu, Taiwan

ARTICLE INFO

Article history:
Received 28 June 2016
Received in revised form 18 October 2016
Accepted 20 October 2016
Available online 24 October 2016
This manuscript was handled by G. Syme, Editor-in-Chief

Keywords:
Stochastic analysis
Probability density function
Stream flow discharge
Lateral inflow

ABSTRACT

This article considers stream flow discharge moving through channels subject to the lateral inflow and described by a linearized diffusion wave equation. The variability of lateral inflow is manifested by random fluctuations in time, which is the only source of uncertainty as to flow discharge quantification. The stochastic nature of stream flow discharge is described by the probability density function (PDF) obtained using the theory of distributions. The PDF of the stream flow discharge depends on the hydraulic properties of the stream flow, such as the wave celerity and hydraulic diffusivity as well as the temporal correlation scale of the lateral inflow rate fluctuations. The focus in this analysis is placed on the influence of the temporal correlation scale and the wave celerity coefficient on the PDF of the flow discharge. The analysis demonstrates that a larger temporal correlation scale causes an increase of PDF of the lateral inflow rate and, in turn, the PDF of the flow discharge which is also affected positively by the wave celerity coefficient.

1. Introduction

Fluctuations in surface lateral inflow into a nearby stream over a large space scale is generally recognized as being affected by natural heterogeneity in surface runoff processes, a by-product of stochastic nature of the rainfall events, rainfall infiltration and ground surface conditions. There are significant lateral inflows contributing to streams during storm-runoff periods in the case of stream reaches with large lateral watershed areas or upslope accumulated areas (Jencso et al., 2009). As such, the stochastic (random) character of the inflow process may lead to large uncertain in the prediction of stream flow discharge based on the deterministic models. The complex nature of the variability in space and time and the limited amount of available field data are among the reasons that motivate the hydrologists to apply the probabilistic concepts to characterize the stream discharge process over large space and time scales (e.g., Abaza et al., 2014; Bonaccorso et al., 2015; Zhao et al., 2015).

The forecasting of stream flow is critical in water quantity and quality managements. For management purposes, it needs to analyze the variations and underlying causes of extreme stream flow discharge events. It is clear that stream flow characterization at a large time scale remains inherent uncertainty due to temporal natural variability of inflow forcing. The uncertainty quantification such as the tails of a probabilistic distribution (or a state variable's PDF) is therefore a prerequisite for making correct decisions.

In most stochastic studies, the second moment (ensemble variance) of the state variable is commonly used to quantify the model uncertainty stemming from heterogeneities in the model parameters. The moment gives a quantitative measure of the error to be anticipated in applying a classical model (large-scale model). However, the measure of uncertainty is not sufficient for the probabilistic risk assessment and management where uncertainty is required to be expressed through the PDF of state variable. Motivated by that, the objective of this work is to quantify uncertainty for flow in lateral inflow-dominated stream channels by the use of a probabilistic model developed by the theory of distributions (e.g., Schwartz, 1952; Kanwal, 1997) within a stochastic framework. Its output such as the PDF of the stream flow discharge will be useful in assessing the rare stream flow events and associated risks. The analysis of results will focus on the impacts of the temporal correlation scale of lateral inflow fluctuations and the hydraulic property of the stream flow (namely, the wave celerity) on the PDF of the stream flow discharge.

To the best of our knowledge, the uncertainty analysis of lateral inflow dominated stream flow on the basis of the diffusion wave model so far has not been presented in the stochastic literature. All the existing studies on quantification of stream flow variability

* Corresponding author at: Institute of Environmental Engineering, National Chiao Tung University, 1001 University Road, Hsinchu 300, Taiwan.

E-mail address: hdyeh@mail.nctu.edu.tw (H.-D. Yeh).

http://dx.doi.org/10.1016/j.jhydrol.2016.10.033
0022-1694/

were carried out based on kinematic wave models with neglecting the effects of inertial and pressure forces (e.g., Scharffenberg and Kavvas, 2011; Ercan and Kavvas, 2012; Wang and Tartakovsky, 2012). Kinematic wave models would be adequate if the flow is unsteady gradually varying and has little backwater effects (Ponce et al., 1978; Singh, 1996). Kinematic waves behave closely to observed natural flood waves in steep rivers with slopes >0.002 (Henderson, 1966). The diffusion wave models (keeping the pressure-gradient term) are an improvement over kinematic wave models because they are capable of accommodating backwater effects (Dooge and Napiorkowski, 1987; Singh, 1996). The diffusion wave approximation is appropriate for simulations of the flood waves in rivers and on flood plains with milder slopes ranging from 0.0001 to 0.001 (Kazezyılmaz-Alhan, 2012). This simplified approximation is therefore appropriate in most cases of practical interest (Ponce, 1989; Singh, 1996; Kazezyılmaz-Alhan, 2012).

A different approach for quantification of stream flow discharge variability to the same problem was taken by Chang and Yeh (2014). The main theme of Chang and Yeh (2014) was to quantify the variability in stream flow discharge in response to fluctuations in lateral inflow rate. In their work, the temporal correlation structure of the fluctuations in the lateral inflow rate was described by the statistics of random fractals. The variance of flow discharge, obtained using the representation theorem, served as an index of large-scale temporal variability. Instead, the focus of the present study is placed on the development of stream flow discharge PDF through the theory of distributions. The random temporal lateral inflow process is characterized by the Langevin equation. When one is concerned with stream flow quantity assessment which involves the probability of rare events, the PDF is required information.

2. Problem formulation

The variations of stream flow discharge Q in response to the temporal changes in lateral inflow rate q_L can be quantified by the linearized diffusion wave equation (e.g., Yen and Tsai, 2001; Fan and Li, 2006) as

$$\frac{\partial}{\partial t}Q(X,t) = D_h\frac{\partial^2}{\partial X^2}Q(X,t) - C_d\frac{\partial}{\partial X}Q(X,t) + C_d q_L(t) \quad 0 < X < L, 0 < t \tag{1}$$

where C_d and D_h represent the wave celerity and hydraulic diffusivity, respectively, and L is the size of the stream flow domain. This study regards the lateral inflow rate as a temporally correlated random function. For a temporally correlated lateral inflow rate and through Eq. (1), the stream flow discharge becomes a temporally correlated function. In other words, the temporal variability of Q is produced by the correlated stochastic process q_L.

It is important to know that Eq. (1) is linearized around the initial steady-state uniform flow condition through the first-order perturbation analysis (e.g., Dooge and Napiorkowski, 1984; Tingsanchali and Manandhar, 1985). As such, Eq. (1) is only accurate to the case where the fluctuations in discharge are small compared with the initial uniform reference discharge and the parameters C_d and D_h in Eq. (1) are related to the initial hydraulic properties and reference flow condition.

Due to temporal variation of lateral inflow processes, the linearized diffusion wave equation (1) is viewed as a stochastic differential equation with stochastic effective parameters (C_d and D_h), a stochastic input (q_L), and therefore a stochastic output (variation of stream flow discharge). As such, C_d and D_h here represent the ensemble means of the wave celerity and hydraulic diffusivity, respectively. In general, both C_d and D_h are closely related to the hydraulic properties of the stream flow, such as the stream channel cross-section geometry, the stream channel bed slope, and the area of the stream flow (e.g., Yen and Tsai, 2001). That is, the stream flow discharge perturbations are dependent of statistic properties of lateral inflow rate and the hydraulic properties of the stream flow.

In the present study, we concentrate on the effects of temporal variability in lateral inflow rate and assume that the stream flow discharge is maintained at the initial uniform value (no perturbation) at the stream boundaries. The flow variability is attributed solely to the stochastic nature of inflow events. The initial and boundary conditions for the variation of stream flow discharge considered here are specified as

$$Q(X,0) = 0 \tag{2}$$

$$Q(0,t) = 0 \tag{3a}$$

$$Q(L,t) = 0 \tag{3b}$$

where L is the size of a bounded domain. Eq. (1) along with Eqs. (2) and (3), constituting a stochastic system of equations, states that for prescribed (deterministic) initial and boundary conditions, the variability of stream flow in space and time is produced only by temporal random fluctuations in lateral inflow rate.

With the aid of Duhamel's principle, the solution to the inhomogeneous problem (Eqs. (1)–(3)) can be found once the solution to the homogeneous version (parameterized by S) is known and expressed as

$$\frac{\partial}{\partial t}Q(X,t;S) = D_h\frac{\partial^2}{\partial X^2}Q(X,t;S) - C_d\frac{\partial}{\partial X}Q(X,t;S) \quad 0 < X < L, S < t \tag{4}$$

$$Q(X,S;S) = C_d q_L(S) \tag{5}$$

$$Q(0,t;S) = 0 \tag{6a}$$

$$Q(L,t;S) = 0 \tag{6b}$$

The solution to $Q(X, t)$ with the source term in Eq. (1) is then given by

$$Q(X,t) = \int_0^t Q(X,t;S)dS \tag{7}$$

Following Pope (1981, 2000) and Tartakovsky and Broyda (2011), the fine-grained PDF of the stream flow discharge can be defined as

$$p(q;X,t;S) = \langle \Theta(q;X,t;S) \rangle \tag{8}$$

where < > designates the ensemble average and

$$\Theta(q;X,t;S) = \delta[Q(X,t;S) - q] \tag{9}$$

where q is an outcome for flow discharge event which will occur. The delta function in Eq. (9) is not an ordinary function in the usual sense but rather a generalized function or distribution in the theory of distributions (e.g., Kanwal, 1997). A functional approach to generalized functions was originally introduced by Schwartz (1952).

In the theory of distributions, a test function such as the delta function can be mapped into real or complex numbers by

$$(\Theta, f) = \int_{-\infty}^{\infty} f(X)\Theta(X)dX \tag{10}$$

where $f(X)$ is a fixed function. The higher-order partial derivatives of a distribution can then be defined applying the functional Eq. (10) and integration by parts. This results in the general relation