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Modeling rainfall-runoff processes using smoothed particle hydrodynamics with mass-varied particles

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ABSTRACT

In this study, a novel treatment of adopting mass-varied particles in smoothed particle hydrodynamics (SPH) is proposed to solve the shallow water equations (SWEs) and model the rainfall-runoff process. Since SWEs have depth-averaged or cross-section-averaged features, there is no sufficient dimension to add rainfall particles. Thus, SPH-SWE methods have focused on modeling discharge flows in open channels or floodplains without rainfall. With the proposed treatment, the application of SPH-SWEs can be extended to rainfall-runoff processes in watersheds. First, the numerical procedures associated with using mass-varied particles in SPH-SWEs are introduced and derived. Then, numerical validations are conducted for three benchmark problems, including uniform rainfall over a 1D flat sloping channel, nonuniform rain falling over a 1D three-slope channel with different rainfall durations, and uniform rainfall over a 2D plot with complex topography. The simulated results indicate that the proposed treatment can avoid the necessity of a source term function of mass variation, and no additional particles are needed for the increase of mass. Rainfall-runoff processes can be well captured in the presence of hydraulic jumps, dry/wet bed flows, and supercritical/subcritical/transcritical flows. The proposed treatment using mass-varied particles was proven robust and reliable for modeling rainfall-runoff processes. It can provide a new alternative for investigating practical hydrological problems.

1. Introduction

The rainfall-runoff process is an important hydrological phenomenon that relates the stream flow response of a river to a given amount of rainfall (Beven, 2001). Rainfall-runoff modeling can be classified into two major categories based on the hydrologic system: black box lumped modeling and physically based distributed modeling (Chow et al., 1988). Lumped modeling averages parameters for the entire watershed, ignores flow-routing mechanisms, and transforms effective rainfall into an outflow hydrograph. It can quickly obtain results, but cannot provide detailed physical processes (Freeze and Harlan, 1969). Distributed modeling considers variations in variables and parameters based on understanding different physical processes. It solves the shallow water equations (SWEs) computationally using the fully dynamic wave approach or the simplified forms of the SWEs using the diffusive wave approximation or the kinematic wave approximation. A variety of numerical methods have been used to solve rainfall-runoff problems, including finite difference methods (Esteves et al., 2000; Fiedler and Ramirez, 2000), finite volume methods (Cea et al., 2010; Costabile et al., 2013), and finite element methods (Vieux and Gauer, 1994). These grid-based methods have provided satisfactory modeling results. However, some physical and numerical challenges still exist, such as those associated with free surfaces, large deformation flows, mixed flow regimes, complex topographies and wet-dry interfaces (Liu and Liu, 2003).

In addition to the aforementioned grid-based methods, some meshless methods have become increasingly popular. Among them, smoothed particle hydrodynamics (SPH) is the most widely adopted (Liu and Liu, 2003). SPH has many useful features for hydrological modeling compared to the traditional grid-based methods. For example, due to its Lagrangian nature, SPH is free of numerical oscillations since there is no convective term (nonlinear term) in the governing equations (Liu and Liu, 2003; Chang et al., 2014), and mass is fully conserved (Vacondio et al., 2012a; Chang et al., 2014). Mixed flow regimes incorporating subcritical, transcritical, and supercritical flows can be efficiently handled (Chang et al., 2011). Complex topographies can be easily evaluated using bottom particles (Vacondio et al., 2012a), and no extra treatment is needed for wet-dry interfaces. Previously, SPH solutions to

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SWEs have focused on modeling discharge flows in open channels or floodplains without rainfall, such as studies that analyzed inflow/outflow boundary conditions (Vacondio et al., 2011), a modified virtual boundary method for imposing closed boundary conditions on arbitrary geometries (Vacondio et al., 2012a), nonrectangular and non-prismatic channels (Chang and Chang, 2013), particle refinement for enhanced resolution (Vacondio et al., 2012b), dam break flows (Chang et al., 2011), floodplain overland flows (Kao and Chang, 2012), and mixed flow regimes at open channel junctions (Chang and Chang, in press). Nevertheless, to the authors' knowledge, studies have not extended SPH-SWE modeling to the rainfall-runoff process.

Some difficulties are associated with using SPH to solve SWEs. Unlike the three-dimensional (3D) Navier-Stokes Equations, SWEs are two-dimensional (2D) depth-averaged equations in the x-y plane or one-dimensional (1D) cross-section-averaged equations (also called the Saint-Venant equations) in the streamwise direction l. Thus, no dimension exists for vertical rainfall inputs. Therefore, mass-varied particles are used in the SPH-SWE in this study to overcome this issue when modeling the model rainfall-runoff process. Fig. 1 illustrates the concept of how mass-varied particles work. In modeling discharge flows without rainfall, river flows are discretized with water slide particles using a 1D SPH method (Chang et al., 2011), while surface overland flows are discretized with water column particles using a 2D SPH method (Kao and Chang, 2012) (both are presented in blue[1] in Fig. 1b). As rainfall occurs, the additional masses are added to the water slides or columns according to the product of their bottom area and the variation in water depth (presented in orange in Fig. 1b). With this new treatment, the SPH-SWE can address mass-varied flow fields, such as rainfall, infiltration and lateral flows, without adding particles. Furthermore, this treatment does not need to construct the source term of mass variation in advance, which is helpful for encoding.

This paper is structured as follows. In Section 2, the SPH-SWE numerical method is briefly introduced, and the derivation of the solving procedure with mass-varied particles for modeling rainfall-runoff process is presented in detail. In Section 3, the numerical scheme is tested using measured rainfall hydrographs at the outlets of 1D channels from Delestre et al. (2009) and Iwagaki (1955) and flow velocity data from a 2D plot with complex topography. The numerical convergence and accuracy are examined and discussed, and conclusions are made in the end.

2. Methodology

2.1. The shallow water equations

The Lagrangian forms of 1D SWEs in Eqs. (1) and (2) and the 2D SWEs in Eqs. (3) and (4) are adopted to address fluid motion in this study.

$$\frac{DA}{Dt} = -A\frac{\partial}{\partial x}\left(\frac{Q}{A}\right) + R \tag{1}$$

$$\frac{DQ}{Dt} = -Q\frac{\partial}{\partial x}\left(\frac{Q}{A}\right) - gA\frac{\partial d_w}{\partial x} + gA(S_0 - S_f) \tag{2}$$

$$\frac{Dd_w}{Dt} = -d_w\nabla\cdot\mathbf{v} + R \tag{3}$$

$$\frac{D\mathbf{v}}{Dt} = -g\nabla d_w + g(-\nabla b - \mathbf{S}_f) \tag{4}$$

In the above, Eqs. (1) and (3) are the continuity equations, while Eqs. (2) and (4) are the momentum equations. In these equations, t is the time, A is the wetted cross-section area, Q is the discharge, d_w is the water depth, R is the rainfall intensity, $\mathbf{v}$ is the horizontal velocity vector (=$\mathbf{v}(u, v)$), u is the x-component of $\mathbf{v}$, v is the y component of $\mathbf{v}$, S_0 is the bed slope, b is the bottom elevation, S_f ($\mathbf{S}_f$) is the friction slope, which is calculated according to either Manning's friction law (Kao and Chang, 2012; Chang and Chang, 2013) or the Darcy-Weisbach friction law (Delestre et al., 2009), and g is the gravitational acceleration. The 1D SWEs govern the wetted cross-section area and the water discharge, and the 2D SWEs govern the water depth and the water velocity.

2.2. Water depth/cross-section wetted area evolution

In SPH, the scale function $f(\mathbf{x})$ can be approximated as follows:

$$f_i = \sum_j^N \frac{m_j}{\rho_j} f_j W_{ij}^i \tag{5}$$

where m_j is the mass of particle j (=$\Delta x_0 \cdot \rho_j$in 1D and $\Delta x_0 \cdot \Delta y_0 \cdot \rho_j$ in 2D); Δx_0 and Δy_0 are the initial particle spacings in the x- and y-directions, respectively; ρ_j is the density of particle j defined as $\rho_w \cdot A$ in 1D and $\rho_w \cdot d_w$ in 2D; ρ_w is the constant water density (1000 kg/m^3); N is the number of particles within the support domain of particle i; W_{ij}^i is the kernel function; and $\nabla_i W_{ij}^i$ is the first derivative of the kernel function.

Based on Eq. (5), the density of particle i can be expressed as follows.

$$\rho_i = \sum_j^N m_j W_{ij}^i \tag{6}$$

To obtain a more accurate solution of the density, the smoothing length is varied based on Eq. (7), which is derived under the assumption of a constant total particle mass within the particle support domain (Springel and Hernquist, 2002):

$$h_i = h_{0,i}\left(\frac{\rho_{0,i}}{\rho_i}\right)^{1/D_m} \tag{7}$$

where $\rho_{0,i}$ and $h_{0,i}$ are the initial density and smoothing length of particle i, respectively, and D_m is the number of spatial dimensions. Eq. (6) thereby becomes nonlinear. We use the Newton-Raphson iteration method to solve Eq. (6) and obtain the density of each particle (Rodriguez-Paz and Bonet, 2005).

2.3. Using mass-varied particles and the modified smoothing length updating formulation

As previously discussed, no extra dimension is available to input rainfall particles when modeling the rainfall-runoff process based on SWEs using traditional methods. However, the rainfall process can be simulated by varying the water depth (which is the particle density ρ) of each particle. Note that if we treat the particle mass as a constant value, the mass and momentum conservation of each particle cannot be achieved simultaneously because the mass and momentum of the raindrop adds to the system. Thus, the particle mass should be varied.

To reflect the rainfall effect on the evolution of the water depth of a fluid particle, we transform the rainfall amount into the increase of the mass of a fluid particle (Δm) based on Eq. (8):

$$\Delta m = \rho\Delta V = \rho_w \Delta d_w \Delta V = \rho_w R \Delta t \Delta V \tag{8}$$

where V is the particle volume (= m/ρ) and Δt is the time step.

However, the use of mass-varied fluid particles violates the assumption of constant mass in Eq. (7). We further assess the

[1] For interpretation of color in Fig. 1, the reader is referred to the web version of this article.