Accepted Manuscript

A journey through loop braid groups

Celeste Damiani

PII: S0723-0869(16)30092-5
DOI: http://dx.doi.org/10.1016/j.exmath.2016.12.003
Reference: EXMATH 25292

To appear in: *Expo. Math.*

Received date: 18 May 2015
Revised date: 2 December 2016

A JOURNEY THROUGH LOOP BRAID GROUPS

CELESTE DAMIANI

Abstract. In this paper we introduce distinct approaches to *loop braid groups*, a generalization of braid groups, and unify all the definitions that have appeared so far in the literature, with a complete proof of the equivalence of these definitions. These groups have in fact been an object of interest in different domains of mathematics and mathematical physics, and have been called, in addition to loop braid groups, with several names such as of *motion groups*, groups of *permutation-conjugacy automorphisms*, *braid-permutation groups*, *welded braid groups* and *untwisted ring groups*. In parallel to this, we introduce an extension of these groups that appears to be a more natural generalization of braid groups from the topological point of view. Throughout the text we motivate the interest in studying loop braid groups and give references to some of their applications.

1. Introduction

Braid groups B_n were introduced by Hurwitz [31] in 1891 as fundamental groups of configuration spaces of n points in the complex plane. However, they owe their name to Artin [1]: he considered them in terms of braid automorphisms of F_n, the free group of rank n, but also in geometric terms. The geometric interpretation certainly is the most intuitive and best known, in particular because of its use in knot theory. Then, Magnus [39] considered braid groups from the point of view of mapping classes, while Markov [40] introduced these groups from a purely group-theoretic point of view. All these points of view have long been known to be equivalent [56]. We can then say that braid groups are ubiquitous objects. Any different definition carries a possible generalization; for instance, we can see braid groups as particular case of Artin-Tits groups, Garside groups, mapping class groups and surface braid groups. Few of these generalizations share with braid groups their principal property: a large family of different equivalent definitions.

Loop braid groups LB_n are a remarkable exception to this fact. Their study has been widely developed during the last twenty years. The first curious fact about these groups is that they appear in the literature with a large number of different names. We choose adopt the terminology introduced by Baez, Wise, and Crans [6], that define loop braid groups in terms of mapping classes. However, these are not the first name and interpretation of loop braids that have appeared in the course of time. In 1986 McCool [41] considers loop braids as *basis-conjugating automorphisms* and in 1996 Savushkina [45] considers them as *permutation conjugacy automorphisms* of F_n, lately called *symmetric automorphisms* by Zaremsky [55]. Loop braids are also known in the literature as *welded braids*, as defined

Date: December 22, 2016.
2010 *Mathematics Subject Classification.* Primary 20F36.
Key words and phrases. Braid groups, welded braid groups, loop braid groups.