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LIMIT CYCLES OF THE CLASSICAL LIÉNARD DIFFERENTIAL SYSTEMS: A SURVEY ON THE LINS NETO, DE MELO AND PUGH'S CONJECTURE

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ABSTRACT. In 1977 Lins Neto, de Melo and Pugh [Lectures Notes in Math. **597**, 335–357] conjectured that the classical Liénard system

$$\dot{x} = y - F(x), \qquad \dot{y} = -x,$$

with $F(x)$ a real polynomial of degree n, has at most $[(n-1)/2]$ limit cycles, where $[\cdot]$ denotes the integer part function. In this paper we summarize what is known and what is still open on this conjecture. For the known results on this conjecture we present a complete proof.

1. Introduction and statement of the main results

The classical Liénard system

$$\dot{x} = y - F(x), \qquad \dot{y} = -x, \tag{1}$$

with $F(x)$ a real polynomial of degree n, has been extensively studied (see for instance [2, 11, 20, 25, 31, 32, 39, 42, 43], and references therein). In 1977 Lins Neto, de Melo and Pugh [25] proved that there exist systems (1) of degree n having $[(n-1)/2]$ limit cycles, and stated the following:

Conjecture *System* (1) *has at most* $[(n-1)/2]$ *limit cycles, where* n *is the degree of the real polynomial* $F(x)$.

Here $[x]$ denotes the integer part function of x.

In this paper we summarize what is known and what is still open on this conjecture. Moreover for the known results on this conjecture we present a complete proof.

The conjecture was based in the following result of Lins Neto, de Melo and Pugh [25]:

Theorem 1. *If the real polynomial* $F(x)$ *has degree* n*, then there are Liénard differential systems* (1) *having at least* $[(n-1)/2]$ *limit cycles.*

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Key words and phrases. Liénard system, limit cycle, conjecture of Lins Neto, de Melo and Pugh.