

Accepted Manuscript

The cardinal Krull dimension of a ring of holomorphic functions

Pete L. Clark

PII: S0723-0869(17)30015-4

DOI: <http://dx.doi.org/10.1016/j.exmath.2017.03.001>

Reference: EXMATH 25294

To appear in: *Expo. Math.*

Received date: 12 June 2016

Please cite this article as: P.L. Clark, The cardinal Krull dimension of a ring of holomorphic functions, *Expo. Math.* (2017), <http://dx.doi.org/10.1016/j.exmath.2017.03.001>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



The Cardinal Krull Dimension of a Ring of Holomorphic Functions

Pete L. Clark

Department of Mathematics, Boyd Graduate Studies Research Center, University of Georgia, Athens, GA 30602 7403, USA
plclark@gmail.com

Abstract

We give an exposition of a recent result of M. Kapovich on the cardinal Krull dimension of the ring of holomorphic functions on a connected \mathbb{C} -manifold. By reducing to the one-dimensional case we give a stronger lower bound for Stein manifolds.

1. Introduction

Recently Kapovich proved the following striking result: if a connected \mathbb{C} -manifold M has a nonconstant holomorphic function, then the ring $\text{Hol}(M)$ of holomorphic functions on M has a chain of prime ideals of continuum length [9].

Here we give an exposition of Kapovich's Theorem, in which the algebraic part of the proof is shortened and simplified. Kapovich defines a class of "ample rings" and shows – by a beautiful use of Sard's Theorem that we follow closely – that if $\text{Hol}(M) \supsetneq \mathbb{C}$ then $\text{Hol}(M)$ is ample, and then he shows that any ample ring has a chain of prime ideals of continuum length [9, Thm. 4]. The proofs, and even the definition of ample rings, make use of hyperreals and hypernatural numbers.

We observe that the analytic part of Kapovich's argument shows that if $\text{Hol}(M) \supsetneq \mathbb{C}$ then $\text{Hol}(M)$ admits an infinite sequence of discrete valuations $\{v_k\}$ which are independent in the sense that for any sequence $\{n_k\}$ of natural numbers there is $f \in \text{Hol}(M)$ such that $v_k(f) = n_k$ for all k , and then we show (Theorem 1.4) that any ring admitting a sequence of independent discrete valuations has a chain of prime ideals of continuum length. In place of nonstandard analysis our proofs use *ultralimits*. In order to understand these, a reader need only know what an ultrafilter is and that an ultrafilter on a compact space converges to a unique point.

Although \mathfrak{c} is a large number, it may not be large enough! Results of Henriksen [7] and Alling [1], [2] show that if M is a noncompact Riemann surface then $\text{Spec Hol}(M)$ has a chain of length 2^{\aleph_1} . Building on these results we show (Theorem 3.3) that for a connected \mathbb{C} -manifold M , if $M \cong V \times N$ for a Stein manifold V of positive dimension, then $\text{Hol}(M)$ admits a chain of prime ideals of length 2^{\aleph_1} .

Download English Version:

<https://daneshyari.com/en/article/5771505>

Download Persian Version:

<https://daneshyari.com/article/5771505>

[Daneshyari.com](https://daneshyari.com)