

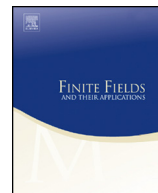


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## On the difference between permutation polynomials



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## ABSTRACT

The well-known Chowla and Zassenhaus conjecture, proved by Cohen in 1990, states that if  $p > (d^2 - 3d + 4)^2$ , then there is no complete mapping polynomial  $f$  in  $\mathbb{F}_p[x]$  of degree  $d \geq 2$ . For arbitrary finite fields  $\mathbb{F}_q$ , a similar non-existence result was obtained recently by Işık, Topuzoğlu and Winterhof in terms of the Carlitz rank of  $f$ .

Cohen, Mullen and Shiue generalized the Chowla–Zassenhaus–Cohen Theorem significantly in 1995, by considering differences of permutation polynomials. More precisely, they showed that if  $f$  and  $f + g$  are both permutation polynomials of degree  $d \geq 2$  over  $\mathbb{F}_p$ , with  $p > (d^2 - 3d + 4)^2$ , then the degree  $k$  of  $g$  satisfies  $k \geq 3d/5$ , unless  $g$  is constant. In this article, assuming  $f$  and  $f + g$  are permutation polynomials in  $\mathbb{F}_q[x]$ , we give lower bounds for the Carlitz rank of  $f$  in terms of  $q$  and  $k$ . Our results generalize the above mentioned result of Işık et al. We also show for a special class of per-

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mutation polynomials  $f$  of Carlitz rank  $n \geq 1$  that if  $f + x^k$  is a permutation over  $\mathbb{F}_q$ , with  $\gcd(k + 1, q - 1) = 1$ , then  $k \geq (q - n)/(n + 3)$ .

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### 1. Introduction

Let  $\mathbb{F}_q$  be the finite field with  $q = p^r$  elements, where  $r \geq 1$  and  $p$  is a prime. Throughout we assume  $q \geq 3$ . We recall that  $f \in \mathbb{F}_q[x]$  is a *permutation polynomial* over  $\mathbb{F}_q$  if it induces a bijection from  $\mathbb{F}_q$  to  $\mathbb{F}_q$ . If  $f(x)$  and  $f(x) + x$  are both permutation polynomials over  $\mathbb{F}_q$ , then  $f$  is called a *complete mapping*. We refer the reader to [11] for a detailed study of complete mapping polynomials over finite fields. Their use in the construction of mutually orthogonal Latin squares is described, for instance, in [9]. For various other applications, see [10,12–14]. Recent work on generalizations of complete mappings can be found in [17].

Theorem 1 below was conjectured by Chowla and Zassenhaus [3] in 1968, and proved by Cohen [4] in 1990.

**Theorem 1.** ([4, Theorem 1]) *If  $d \geq 2$  and  $p > (d^2 - 3d + 4)^2$ , then there is no complete mapping polynomial of degree  $d$  over  $\mathbb{F}_p$ .*

A significant generalization of this result was obtained by Cohen, Mullen and Shiue [5] in 1995, and gives a lower bound for the degree of the difference of two permutation polynomials in  $\mathbb{F}_p[x]$  of the same degree  $d$ , when  $p > (d^2 - 3d + 4)^2$ .

**Theorem 2.** ([5, Theorem 2]) *Suppose  $f$  and  $f + g$  are monic permutation polynomials over  $\mathbb{F}_p$  of degree  $d \geq 3$ , where  $p > (d^2 - 3d + 4)^2$ . Then either  $\deg(g) = 0$  or  $\deg(g) \geq 3d/5$ .*

An alternative invariant, the so-called Carlitz rank, attached to permutation polynomials, was recently used by Işık, Topuzoğlu and Winterhof [8] to obtain a non-existence result, similar to that in Theorem 1. The concept of Carlitz rank was first introduced in [1]. We describe it here briefly. The interested reader may see [16] for details.

By a well-known result of Carlitz [2] that any permutation polynomial over  $\mathbb{F}_q$ ,  $q \geq 3$ , is a composition of linear polynomials  $ax + b$ ,  $a, b \in \mathbb{F}_q$ ,  $a \neq 0$ , and  $x^{q-2}$ , any permutation  $f$  over  $\mathbb{F}_q$  can be represented by a polynomial of the form

$$P_n(x) = \left( \dots \left( (a_0x + a_1)^{q-2} + a_2 \right) \dots + a_n \right)^{q-2} + a_{n+1}, \tag{1.1}$$

for some  $n \geq 0$ , where  $a_i \neq 0$ , for  $i = 0, 2, \dots, n$ . Note that  $f(c) = P_n(c)$  holds for all  $c \in \mathbb{F}_q$ , however this representation is not unique, and  $n$  is not necessarily minimal.

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