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A complete characterization of Galois subfields of the generalized Giulietti–Korchmáros function field

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ABSTRACT

We give a complete characterization of all Galois subfields of the generalized Giulietti–Korchmáros function fields $\mathcal{C}_n/\mathbb{F}_{q^{2n}}$ for $n \geq 5$. Calculating the genera of the corresponding fixed fields, we find new additions to the list of known genera of maximal function fields.

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1. Introduction

Let F be a function field of genus $g(F)$ over the finite field $\mathbb{F}_\ell$ with ℓ elements. The Hasse–Weil theorem gives the following upper bound for the number of rational places $N(F)$ of F:

$$N(F) \le \ell + 1 + 2g(F)\sqrt{\ell}\ .$$

Function fields attaining this bound are called maximal, and have played a central role in the theory of function fields over finite fields (or equivalently curves over finite fields).

An important example of maximal function fields is the Hermitian function field $\mathcal{H}$ over the finite field $\mathbb{F}_{q^2}$, where q is a power of a prime p. It is given by $\mathcal{H} = \mathbb{F}_{q^2}(x, y)$ with

$$x^q + x = y^{q+1}\ .$$

It has genus $q(q-1)/2$ (in fact the largest possible genus for a maximal function field over $\mathbb{F}_{q^2}$) and a large automorphism group $A \cong \mathrm{P}GU(3, q)$. By a theorem of Serre (see [14]) a subfield of a maximal function field is itself also maximal, provided it is a function field over the same field of constants. Studying such subfields of the Hermitian function field leads to many new examples of maximal function fields. One way to construct such subfields is by taking fixed fields of subgroups of A (see among others [1,2,8] and the references in [7]). Since all maximal subgroups of A are known, interest has been diverted into studying subgroups of the various maximal subgroups. The maximal subgroup $A(P_\infty)$ fixing the unique pole P_∞ of x, together with an involution generates the whole automorphism group A. A complete characterization of all subgroups of $A(P_\infty)$ and the genera of the corresponding fixed fields has been given in [2].

For a long time, all known maximal function fields were subfields of the Hermitian function field. This led to the question whether any maximal function fields could be embedded as subfields in the Hermitian function field. Giulietti and Korchmáros [10] introduced a new family of maximal function fields (GK function fields) over finite fields $\mathbb{F}_{q^6}$, which are not subfields of the Hermitian function field over the corresponding field for $q > 2$. The GK function field is given by $\mathcal{C} = \mathbb{F}_{q^6}(x, y, z)$ with

$$x^q + x = y^{q+1} \quad \text{and} \quad z^{(q^3+1)/(q+1)} = y \sum_{i=0}^{q} (-1)^{i+1} x^{i(q-1)}\ .$$

The GK function field was generalized in [9] to a family of maximal function fields over finite fields $\mathbb{F}_{q^{2n}}$ with n odd. The generalized GK (GGK) function field, also known as the Garcia–Güneri–Stichtenoth function field, is given by $\mathcal{C}_n = \mathbb{F}_{q^{2n}}(x, y, z)$ with

$$x^q + x = y^{q+1} \quad \text{and} \quad z^{(q^n+1)/(q+1)} = y^{q^2} - y\ .$$