Finite Fields and Their Applications 48 (2017) 371–394

Contents lists available at ScienceDirect

Finite Fields and Their Applications

www.elsevier.com/locate/ffa

Locality of optimal binary codes

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ARTICLE INFO

Article history:
Received 29 October 2016
Received in revised form 1 July 2017
Accepted 27 August 2017
Available online xxxx
Communicated by Chaoping Xing

MSC:
11T71
14G50
94B05

Keywords:
Locally repairable code
Repair locality
MacDonald code
Optimal code

ABSTRACT

The locality of locally repairable codes (LRCs) for a distributed storage system is the number of nodes that participate in the repair of failed nodes, which characterizes the repair cost. In this paper, we first determine the locality of MacDonald codes, then propose three constructions of LRCs with $r = 1, 2$ and 3. Based on these results, for $2 \leq k \leq 7$ and $n \geq k + 2$, we give an optimal linear $[n, k, d]$ code with small locality. The distance optimality of these linear codes can be judged by the codetable of M. Grassl for $n < 2(2^k - 1)$ and by the Griesmer bound for $n \geq 2(2^k - 1)$. Almost all the $[n, k, d]$ codes ($2 \leq k \leq 7$) have locality $r \leq 3$ except for the three codes, and most of the $[n, k, d]$ code with $n < 2(2^k - 1)$ achieves the Cadambe–Mazumdar bound for LRCs.

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1. Introduction

For a distributed storage system (DSS), replication is the simplest and most widespread technique to ensure resilience against node failures, this strategy decreases the storage efficiency. Locally repairable codes are a family of erasure codes that can

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http://dx.doi.org/10.1016/j.ffa.2017.08.013
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recover any code symbol by accessing other survived code symbols. This will enable to recover the data stored in failed nodes locally, by accessing other nodes in the system.

LRCs were introduced by Gopalan et al. in [1], some basic properties of LRCs were also discussed and a Singleton-like bound on the distance d of an (n, k, r) was proved in [1]:

$$d \leq n - k + 2 - \lceil \frac{k}{r} \rceil. \tag{1}$$

If $r = k$, the bound (1) degenerates to the classical Singleton bound. When considering the size of the alphabet of field, a new bound called Cadambe–Mazumdar (CM) bound was established in [2], it says that an $[n, k, d]$ code with locality r satisfies

$$k \leq min_{t \in \mathbb{Z}^+} \{tr + k_{opt}^{(q)}(n - t(r + 1), d)\}, \tag{2}$$

where $k_{opt}^{(q)}(n, d)$ is the largest possible dimension of a code of length n, for given field size q and minimum distance d. A more general bound considering the availability t is presented in [9].

There exist lots of work devoted to constructions of LRCs attaining the bound (1), such as in [1,11–13,15,16] and references therein. Except for another proof of the general Singleton bound (1), using polynomial interpolation to recover the lost symbol from r known symbols in its recovery set, a construction of optimal LRCs over finite field $q > n$ was given in [12]. With a different approach from Construction (I) in [7] of the case $(r + 1)|n$, Tamo et al. [13] used an $[r + 1, r]$ MDS code to re-encode the Reed–Solomon code and constructed a family of LRCs with optimal minimum distance. However, these LRCs are all constructed over large finite field F_q, where q is an exponential function of n or greater than code length n. In practice, considering fast arithmetic and backward compatibility with existing hardware, codes designed over binary field is the best choice [7].

In this paper, we focus on construction of binary LRCs and determine their locality. Before introducing our work, we simply introduce known results on the construction of LRCs over the binary field F_2.

Refs. [4,5,7,8,10,14,22,23] propose construction of optimal LRCs over small field. Based on different base codes (systematic binary codes), Huang et al. [4] proposed constructions of binary LRCs with $d = 3, 4$ and 5 using phantom parity-check symbols, the codes with $d = 4$ are optimal. Silberstein et al. used anticodes and modified a binary Simplex code to construct new LRCs attaining the bound (2) with locality $r = 2, 3$ and most of the LRCs constructed attain the Griesmer bound [5]. Goparaju and Calderbank [7] presented binary cyclic LRCs with optimal dimension for distances 2, 6 and 10 and locality 2. Several binary codes (binary reversible code, binary code with Simplex locality) were derived from a locality code in [8]. Song et al. [10] constructed a binary $[n = (r + 1)^m, k = r^m]$ code with availability $t = 2^m - 1$ that is equivalent to the direct product of m copies of the $[r + 1, r]$ single-parity-check code. Tamo et al. presented some binary LRCs

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