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Weierstrass semigroups from Kummer extensions ☆

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The Weierstrass semigroups and pure gaps can be helpful in constructing codes with better parameters. In this paper, we investigate explicitly the minimal generating set of the Weierstrass semigroups associated with several totally ramified places over arbitrary Kummer extensions. Applying the techniques provided by Matthews in her previous work, we extend the results of specific Kummer extensions studied in the literature. Some examples are included to illustrate our results.

1. Introduction

Since Goppa [1] constructed algebraic geometric (AG) codes from several rational places, the study of AG codes becomes an important instrument in coding theory. For a given AG code, the famous Riemann–Roch theorem gives a non-trivial lower bound,

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named Goppa bound, for the minimum distance in a very general setting [2,3]. Garcia, Kim and Lax improved the Goppa bound using the arithmetical structure of Weierstrass gaps at one place in [4,5]. Homma and Kim [6] introduced the concept of pure gaps and demonstrated a similar result for a divisor concerning a pair of places. And this was generalized to several places by Carvalho and Torres in [7].

Weierstrass semigroups and pure gaps are of significant uses in the construction and analysis of AG codes. They would be applied to obtain codes with better parameters. The authors of [8–10] computed the Weierstrass gap sets and improved the parameters of one-point AG codes from Hermitian curves. For multi-point AG codes from Kummer extensions, see [11–13] for example.

Weierstrass semigroups over specific Kummer extensions were well-studied in the literature. For instance, Matthews [14] investigated the Weierstrass semigroup of any l-tuple of collinear places on a Hermitian curve. In [15], Matthews generalized the results of [7,14] by determining the Weierstrass semigroup of any l-tuple rational places on the quotient of the Hermitian curve defined by the equation $x^q + x = y^m$ over $\mathbb{F}_{q^2}$ where $m > 2$ is a divisor of $q + 1$. However, little is known for general Kummer extensions, except that, Castellanos, Masuda and Quoos [12] recently described the Weierstrass semigroups of one and two rational places.

In this paper, our main interest will be in the research of the Weierstrass semigroup of any l-tuple rational places over arbitrary Kummer extensions. This work is strongly inspired by the study of [12,14,15]. We shall explicitly calculate the minimal generating set of the Weierstrass semigroups by employing the techniques provided by Matthews in [14,15]. At this point, we extend the results of [12,14,15]. We mention that our results can be employed to get linear codes attaining good or better records on the parameters, see [11–13] for more details.

The remainder of the paper is organized as follows. In Section 2 we briefly recall some notations and preliminary results over arbitrary function fields. Section 3 focuses on the determination of the minimal generating sets of the Weierstrass semigroups over Kummer extensions. Finally, in Section 4 we exhibit some examples by using our main results.

2. Preliminaries

In this section, we introduce notations and present some basic facts on the minimal generating set of Weierstrass semigroups of distinct rational places over arbitrary function fields.

Let q be a power of a prime p and $\mathbb{F}_q$ be a finite field with q elements. We denote by F a function field with genus g over $\mathbb{F}_q$ and by $\mathbb{P}_F$ the set of places of F. The free abelian group generated by the places of F is denoted by $\mathcal{D}_F$, whose element is called a divisor. Assume that $D = \sum_{P\in\mathbb{P}_F} n_P P$ is a divisor such that almost all $n_P = 0$, then the degree of D is $\deg(D) = \sum_{P\in\mathbb{P}_F} n_P$. For a function $f \in F$, the divisor of f will be denoted by (f) and the divisor of poles of f will be denoted by $(f)_\infty$.