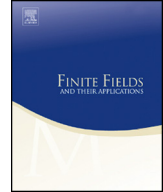




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## Finite Fields and Their Applications

[www.elsevier.com/locate/ffa](http://www.elsevier.com/locate/ffa)
 $G_2$  and some exceptional Witt vector identities

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## ABSTRACT

We find some new one-parameter families of exponential sums in every odd characteristic whose geometric and arithmetic monodromy groups are  $G_2$ .

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## 0. Introduction

In earlier work [7, 9.1.1], we proved that certain very simple one-parameter families of exponential sums had the exceptional group  $G_2$  as their (geometric and arithmetic) monodromy groups, in every finite characteristic  $p \geq 17$ . These sums were of the form

$$(1/g) \sum_{x \in k^\times} \chi_2(x) \psi(x^7 + tx).$$

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Here  $k$  is a finite field,  $g$  is a fixed gauss sum,  $\chi_2$  is the quadratic character of  $k^\times$ ,  $\psi$  is a nontrivial additive character of  $k$ , and  $t \in k$  is the parameter. A question of Rudnick and Waxman led us to wonder if, in this construction, the polynomial  $x^7$  inside the  $\psi$  could be replaced by other polynomials of degree seven and still yield  $G_2$ . Computer experiments suggested that the answer was indeed yes, for polynomials of the form

$$ax^7/7 + 2abx^5/5 + ab^2x^3/3,$$

any  $a \neq 0$ , any  $b$ . That these polynomials do indeed produce  $G_2$  in large characteristic (see [Theorem 4.3](#)) results from certain Witt vector identities. It remains an open question if these are the only polynomials which produce  $G_2$ .

In the second half of the paper, we analyze the situation in low characteristic, especially in characteristics 3, 5, 7, where Witt vectors reappear in order to make sense of the question, and (again) to provide the answer.

## 1. The exceptional identities

Fix a prime  $p$ , and consider the  $p$ -Witt vectors of length 2 as a ring scheme over  $\mathbb{Z}$ . The addition law is given by

$$(x, a) + (y, b) := (x + y, a + b + (x^p + y^p - (x + y)^p)/p).$$

The multiplication law is given by

$$(x, a)(y, b) := (xy, x^pb + y^pa + pab).$$

For an odd prime  $p$ , we have

$$(x, 0) + (y, 0) + (-x - y, 0) = (0, (x^p + y^p - (x + y)^p)/p).$$

Let us define, for odd  $p$ , the integer polynomial

$$F_p(x, y) := (x^p + y^p - (x + y)^p)/p \in \mathbb{Z}[x, y].$$

For  $p = 2$ , we have

$$(x, 0) + (y, 0) + (-x - y, 0) = (0, x^2 + xy + y^2),$$

and we define

$$F_2(x, y) := x^2 + xy + y^2 \in \mathbb{Z}[x, y].$$

Thus

$$F_3 = -xy(x + y).$$

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