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On maximal cliques of polar graphs ☆

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A B S T R A C T

The maximal cliques of the graph $\mathrm{NU}(4, q^2)$ related to the Hermitian surface of $\mathrm{PG}(3, q^2)$ and of the graph $\mathrm{NO}^{\pm}(2n+2, q)$, q even, $n \geq 1$, are classified.

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1. Introduction

Let $\mathcal{Q}^{\pm}(2n+1,q)$ be a non-degenerate quadric of $\mathrm{PG}(2n+1,q)$, $n \geq 1$. Let $\mathrm{NO}^{\pm}(2n+2,q)$ be the graph whose vertices are the points of $\mathrm{PG}(2n+1,q) \setminus \mathcal{Q}^{\pm}(2n+1,q)$ and two vertices P_1, P_2 are adjacent if the line joining P_1 and P_2 contains exactly one point of $\mathcal{Q}^{\pm}(2n+1,q)$ (i.e., it is a line tangent to $\mathcal{Q}^{\pm}(2n+1,q)$). The graph $\mathrm{NO}^{\pm}(2n+2,q)$ is a strongly regular graph if and only if $q = 2$ [1, Table 9.9, p. 145]. Let $\mathcal{H}(n,q^2)$ be a non-degenerate Hermitian variety of $\mathrm{PG}(n,q^2)$, $n \geq 2$. Let $\mathrm{NU}(n+1,q^2)$ be the graph whose vertices are the points of $\mathrm{PG}(n,q^2) \setminus \mathcal{H}(n,q^2)$ and two vertices P_1, P_2 are adjacent if the line joining P_1 and P_2 contains exactly one point of $\mathcal{H}(n,q^2)$ (i.e., it is a line tangent to $\mathcal{H}(n,q^2)$). The graph $\mathrm{NU}(n+1,q^2)$ is a strongly regular graph for any q [1, Table 9.9, p. 145].

The maximal cliques of the graphs $\mathrm{NU}(3,q^2)$ were classified in [2, Corollary 3]. In particular the authors showed that there are exactly two types of cliques of size q^2 and $q+2$, respectively. In the first case, it corresponds to the points on a tangent line with the tangent point excluded. In the second case, it corresponds to $q+1$ points forming a Baer subline of a tangent line ℓ, and the other point not on ℓ.

In this paper we will classify the maximal cliques of the graph $\mathrm{NU}(4,q^2)$ related to the Hermitian surface of $\mathrm{PG}(3,q^2)$ and of the graph $\mathrm{NO}^{\pm}(2n+2,q)$, q even, $n \geq 1$ (cf. Theorem 3.1). In particular we will prove the following classification theorem.

Theorem 1.1. *Let $\mathcal{C}$ be a maximal clique of $\mathrm{NU}(4,q^2)$, then either*

1. $|\mathcal{C}| = q^2$, *and $\mathcal{C}$ consists of the points on a tangent line without the tangent point itself;*

or $\mathcal{C}$ spans the whole space and one of the following cases occur:

2. $|\mathcal{C}| = q+4$, $q > 2$, *and $\mathcal{C}$ consists of a Baer subline and a triangle;*
3. $|\mathcal{C}| = 2q+2$, $q \equiv 1 \pmod 3$ *and $\mathcal{C}$ consists of two Baer sublines on two skew lines;*
4. $|\mathcal{C}| = 2q+3$, $q \not\equiv 1 \pmod 3$ *and $\mathcal{C}$ consists of two Baer sublines on two skew lines and a further point not on the lines;*
5. *$\mathcal{C}$ is an arc of* $\mathrm{PG}(3,q^2)$.

2. The maximal cliques of $\mathrm{NU}(4,q^2)$

The graph $\mathrm{NU}(4,q^2)$ is a strongly regular graph with parameters $v = q^3(q^2+1)(q-1)$, $k = (q^2-1)(q^3+1)$, $\lambda = q^4+q^2-2$ and $\mu = q(q^2-1)(q+1)$.

We first recall that an *O'Nan configuration* consists of six points in a projective plane, namely the four vertices of a non-degenerate quadrangle together with two of their three diagonal points. In [14] it has been proved that no O'Nan configuration lies on a Hermitian curve. By using the polarity of the Hermitian curve, we have the following remark.