

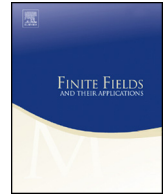


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# On the next-to-minimal weight of affine cartesian codes

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## ABSTRACT

In this paper we determine many values of the second least weight of codewords, also known as the next-to-minimal Hamming weight, for a type of affine variety codes, obtained by evaluating polynomials of degree up to  $d$  on the points of a cartesian product of  $n$  subsets of a finite field  $\mathbb{F}_q$ . Such codes firstly appeared in a work by O. Geil and C. Thomsen (see [12]) as a special case of the so-called weighted Reed–Muller codes, and later appeared independently in a work by H. López, C. Rentería-Marquez and R. Villarreal (see [16]) named as affine cartesian codes. Our work extends, to affine cartesian codes, the results obtained by Rolland in [17] for generalized Reed–Muller codes.

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## 1. Introduction

Let  $K := \mathbb{F}_q$  be a field with  $q$  elements and let  $A_1, \dots, A_n$  be a collection of non-empty subsets of  $K$ . Consider an *affine cartesian set*

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$$\mathcal{X} := A_1 \times \cdots \times A_n := \{(a_1 : \cdots : a_n) \mid a_i \in A_i \text{ for all } i\} \subset \mathbb{A}^n,$$

where  $\mathbb{A}^n$  is the  $n$ -dimensional affine space defined over  $K$ .

For a nonnegative integer  $d$  write  $K[\mathbf{X}]_{\leq d}$  for the  $K$ -vector space formed by the polynomials in  $K[X_1, \dots, X_n]$  of degree up to  $d$  together with the zero polynomial. We denote by  $d_i$  the cardinality of  $A_i$ , for  $i = 1, \dots, n$ . Clearly  $|\mathcal{X}| = \prod_{i=1}^n d_i =: \tilde{m}$  and let  $P_1, \dots, P_{\tilde{m}}$  be the points of  $\mathcal{X}$ . Define  $\phi_d : K[\mathbf{X}]_{\leq d} \rightarrow K^{\tilde{m}}$  as the evaluation morphism  $\phi_d(g) = (g(P_1), \dots, g(P_{\tilde{m}}))$ .

**Definition 1.1.** The image  $C_{\mathcal{X}}(d)$  of  $\phi_d$  is a vector subspace of  $K^{\tilde{m}}$  called the *affine cartesian code* (of order  $d$ ) defined over the sets  $A_1, \dots, A_n$ .

In the special case where  $A_1 = \cdots = A_n = K$  we have the well-known generalized Reed–Muller code of order  $d$ . An affine cartesian code is a type of affine variety code, as defined in [10]. These codes firstly appeared in a work by O. Geil and C. Thomsen (see [12]) as a special case of the so-called weighted Reed–Muller codes, and later appeared independently in a work by H. López, C. Rentería-Marquez and R. Villarreal (see [16]) named as affine cartesian codes. In [16] the authors prove that we may ignore, in the cartesian product, sets with just one element and moreover may always assume that  $2 \leq d_1 \leq \cdots \leq d_n$ . The minimum distance of these codes was found independently by Geil et al. (see [12]) and López et al. (see [16]) and a formula for their dimension was presented in [16].

**Theorem 1.2.** [12, Prop. 5 (with  $w = 1$ )] and [16, Thm. 3.1 and Thm. 3.8]

1) The dimension of  $C_{\mathcal{X}}(d)$  is  $\tilde{m}$  (i.e.  $\phi_d$  is surjective) if  $d \geq \sum_{i=1}^n (d_i - 1)$ , and for  $0 \leq d < \sum_{i=1}^n (d_i - 1)$  we have

$$\begin{aligned} \dim(C_{\mathcal{X}}(d)) &= \binom{n+d}{d} - \sum_{i=1}^n \binom{n+d-d_i}{d-d_i} + \cdots + \\ &(-1)^j \sum_{1 \leq i_1 < \cdots < i_j \leq n} \binom{n+d-d_{i_1}-\cdots-d_{i_j}}{d-d_{i_1}-\cdots-d_{i_j}} + \cdots + (-1)^n \binom{n+d-d_1-\cdots-d_n}{d-d_1-\cdots-d_n} \end{aligned}$$

where we set  $\binom{a}{b} = 0$  if  $b < 0$ .

2) The minimum distance  $\delta_{\mathcal{X}}(d)$  of  $C_{\mathcal{X}}(d)$  is 1, if  $d \geq \sum_{i=1}^n (d_i - 1)$ , and for  $0 \leq d < \sum_{i=1}^n (d_i - 1)$  we have

$$\delta_{\mathcal{X}}(d) = (d_{k+1} - \ell) \prod_{i=k+2}^n d_i$$

where  $k$  and  $\ell$  are uniquely defined by  $d = \sum_{i=1}^k (d_i - 1) + \ell$  with  $0 \leq \ell < d_{k+1} - 1$  (if  $k + 1 = n$  we understand that  $\prod_{i=k+2}^n d_i = 1$ , and if  $d < d_1 - 1$  then we set  $k = 0$  and  $\ell = d$ ).

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