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Perfect codes in poset spaces and poset block spaces

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The paper begins by giving a counter example to show that the algorithm for construction of new perfect poset codes from a given perfect poset code by removal of a coordinate as given by Lee (2004) [11] does not hold. The algorithm has been improved and generalized to obtain new perfect poset block codes from a given perfect poset block code. The modified necessary and sufficient conditions for the construction of new perfect poset codes have been derived as a particular case. A bound has been obtained on the height of poset P_s that turns a given π-code into r-perfect (P_s, π)-code. We show that there does not exist a poset which admits the binary Simplex code of order 3 to be a 2-perfect poset code. Also, all the poset structures which admit the extended ternary Golay code to be a 3-perfect poset code have been classified.

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1. Introduction

The theory of error-correcting codes deals with the study of linear subspaces of $\mathbb{F}_q^n$, where $\mathbb{F}_q^n$ denotes the space of all n-tuples over the finite field $\mathbb{F}_q$. Most of the studies in coding theory have been made on codes endowed with Hamming metric. An optimal class of codes having interesting mathematical structure is the class of perfect codes. A code is said to be perfect if balls of some particular radius centered at the codewords of C are disjoint and cover $\mathbb{F}_q^n$. Perfect codes find applications in group theory, combinatorial configurations and Diophantine number theory. One of the limitations of considering Hamming metric is the scarcity of perfect codes [14].

During 1990's, applications of codes in cryptography, experimental designs and high-dimensional numerical integration and more interestingly, the generalizations of the classical problems of coding theory motivated the researchers to explore codes endowed with metric other than Hamming metric. Using partially ordered sets and Niederreiter's generalization of the classical problem of coding theory [12], Brualdi et al. [2] introduced the concept of codes endowed with poset metric. The class of perfect codes with respect to poset metric is extremely large due to the increased packing radius in this case [5]. Moreover, MDS codes with respect to poset metric also occur in abundance [8]. In [7, 9,10], the approach has been to define poset on the set of n coordinates such that the given code in Hamming space turns out to be a perfect poset code. However, to the best of our knowledge, no general construction is known. In [11], Lee has given an algorithm to add or remove coordinates from a given perfect poset code resulting in new perfect codes. However, the result needs to be revised in view of the counter example given in section 3.

Block metric or π-metric, introduced by Feng et al. [6], is another generalization of Hamming metric. Encompassing poset and block structures, poset block metric was introduced by Alves et al. [1] and is by far the most generalized of all approaches, including the widely studied NRT block metric [13] which forms a subclass. It may be seen that the relationship between block metric and poset block metric is analogous to that of Hamming metric and poset metric. We explore the perfectness of block codes in poset block space.

In section 2, basic definitions have been presented. In section 3, we present a counter example to show that the algorithm given by Lee [11, Theorem 3.1(2)] does not hold. Generalizing the same, a modified algorithm to add or remove blocks from a given poset block metric so as to obtain new perfect codes has been derived. As a particular case, the corresponding necessary and sufficient conditions for the construction of new perfect poset codes have been obtained. In section 4, we present a bound on the height of poset which turns a given π-code into r-perfect poset block code. Using the above, we show that the binary Simplex code of order 3 can not be 2-perfect code with respect to poset metric. Also, all the poset structures which admit the extended ternary Golay code to be a 3-perfect poset code have been classified.