

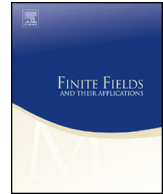


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## Supercongruences involving dual sequences ☆



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## ABSTRACT

In this paper we study some sophisticated supercongruences involving dual sequences. For  $n = 0, 1, 2, \dots$  define

$$d_n(x) = \sum_{k=0}^n \binom{n}{k} \binom{x}{k} 2^k$$

and

$$s_n(x) = \sum_{k=0}^n \binom{n}{k} \binom{x}{k} \binom{x+k}{k} = \sum_{k=0}^n \binom{n}{k} (-1)^k \binom{x}{k} \binom{-1-x}{k}.$$

For any odd prime  $p$  and  $p$ -adic integer  $x$ , we determine  $\sum_{k=0}^{p-1} (\pm 1)^k d_k(x)^2$  and  $\sum_{k=0}^{p-1} (2k+1) d_k(x)^2$  modulo  $p^2$ ; for example, we establish the new  $p$ -adic congruence

$$\sum_{k=0}^{p-1} (-1)^k d_k(x)^2 \equiv (-1)^{\langle x \rangle_p} \pmod{p^2},$$

where  $\langle x \rangle_p$  denotes the least nonnegative integer  $r$  with  $x \equiv r \pmod{p}$ . For any prime  $p > 3$  and  $p$ -adic integer  $x$ , we determine  $\sum_{k=0}^{p-1} s_k(x)^2$  modulo  $p^2$  (or  $p^3$  if  $x \in \{0, \dots, p-1\}$ ), and show that

$$\sum_{k=0}^{p-1} (2k+1) s_k(x)^2 \equiv 0 \pmod{p^2}.$$

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We also pose several related conjectures.

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## 1. Introduction

For a sequence of numbers  $a_0, a_1, a_2, \dots$ , its *dual sequence*  $a_0^*, a_1^*, a_2^*, \dots$  is given by

$$a_n^* = \sum_{k=0}^n \binom{n}{k} (-1)^k a_k \quad (n = 0, 1, 2, \dots). \quad (1.1)$$

It is well-known that  $(a_n^*)^* = a_n$  for all  $n = 0, 1, 2, \dots$ . One may consult [15] for some combinatorial identities involving dual sequences. The author [21, Theorem 2.2] showed that if  $p$  is an odd prime,  $a_0, a_1, \dots, a_{p-1}$  are  $p$ -adic integers and  $m$  is an integer with  $p \nmid m(m-4)$  then

$$\sum_{k=0}^{p-1} \frac{\binom{2k}{k} a_k^*}{(4-m)^k} \equiv \left( \frac{m(m-4)}{p} \right) \sum_{k=0}^{p-1} \frac{\binom{2k}{k} a_k}{m^k} \pmod{p},$$

where  $\left(\frac{\cdot}{p}\right)$  denotes the Legendre symbol.

Let  $p$  be any odd prime. There are various interesting  $p$ -adic congruences related to finite fields, see, e.g., [1,3,7,22,24]. The author and R. Tauraso [23, (1.9)] showed that

$$\sum_{k=0}^{p-1} \binom{2k}{k} \equiv \left(\frac{p}{3}\right) \pmod{p^2},$$

where  $(-)$  is the Legendre symbol. In [17] the author determined  $\sum_{k=0}^{p-1} \binom{2k}{k} / m^k$  modulo  $p^2$  for any integer  $m \not\equiv 0 \pmod{p}$ , and moreover he proved that

$$\sum_{k=0}^{p-1} \frac{\binom{2k}{k}}{2^k} \equiv \left(\frac{-1}{p}\right) - p^2 E_{p-3} \pmod{p^3},$$

where  $E_0, E_1, E_2, \dots$  are the Euler numbers given by

$$E_0 = 1 \quad \text{and} \quad \sum_{\substack{k=0 \\ 2 \nmid k}}^n \binom{n}{k} E_{n-k} = 0 \quad \text{for } n \in \mathbb{Z}^+ = \{1, 2, 3, \dots\}.$$

If  $a_n = \binom{2n}{n} = \binom{-1/2}{n} (-4)^n$  for  $n \in \mathbb{N} = \{0, 1, 2, \dots\}$ , then we have  $a_n^* = (-1)^n T_n$  for all  $n \in \mathbb{N}$  by [4, (3.86)], where the central trinomial coefficient  $T_n$  is the constant term of  $(1+x+x^{-1})^n$ . In [20] the author showed that

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