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Short Communication

A note on character sums in finite fields

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ABSTRACT

We prove a character sum estimate in $\mathbb{F}_q[t]$ and answer a question of Shparlinski.

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1. Introduction

Shparlinski [5] asks the following.

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Problem 1. [5, Problem 14] Let $m \in \mathbb{Z}$ and χ be a non-principal character modulo m. Under the Generalized Riemann Hypothesis, obtain an estimate of the form

$$\sum_{k=1}^{N} \chi(k) \ll N^{1/2} m^{o(1)}$$

for any N, with an explicit expression for $m^{o(1)}$.

Though it has never been explicitly written down, such a bound is presumably well-known among analytic number theorists due to its connection with upper bounds for L-functions. Indeed, let $L(s,\chi)$ be the Dirichlet L-function associated with χ and $s = \sigma + it$. First we assume that χ is primitive. Then conditionally under GRH, we have the bound [3, Exercise 8, Section 13.2]

$$L(s,\chi) \ll \exp\left(C_1 \frac{\log m|t|}{\log\log m|t|} \right) \tag{1}$$

for some absolute constant $C_1 > 0$ and uniformly for $1/2 \le \sigma \le 3/2$ and $|t| \ge 2$. Using the bound (1) and a standard contour integral one can show that

$$\sum_{k=1}^{N} \chi(k) \ll N^{1/2} \exp\left(C_2 \frac{\log m}{\log\log m} \right) \tag{2}$$

for some absolute constant $C_2 > 0$, when χ is primitive. If χ is induced by a character χ_1 modulo r with $r|m$ then

$$\sum_{k=1}^{N} \chi(k) = \sum_{\substack{k=1 \\ (k,r)=1}}^{N} \chi_1(k) = \sum_{d|\frac{m}{r}} \mu(d)\chi_1(d) \sum_{k \le N/d} \chi_1(k).$$

Bounding this trivially, together with the fact that the number of prime factors of n is $O\left(\frac{\log n}{\log\log n}\right)$, it follows that we have a bound of type (2) as well when χ is not primitive.

The purpose of this note, however, is to obtain a bound similar to (2) in the polynomial ring $\mathbb{F}_q[t]$. Let $Q \in \mathbb{F}_q[t]$ be a polynomial of degree n. A (Dirichlet) character χ modulo Q is a character on the multiplicative group $(\mathbb{F}_q[t]/(Q))^\times$, which can be extended to a function on all of $\mathbb{F}_q[t]$ by periodicity and by setting $\chi(f) = 0$ for all $(f,Q) \ne 1$. If Q is irreducible then χ is a character on the field $\mathbb{F}_{q^n}$.

Shparlinski (private communication) also asks an $\mathbb{F}_q[t]$-analog of Problem 1:

Problem 2. Let $Q \in \mathbb{F}_q[t], \deg Q = n > 0$ and χ be a non-principal character modulo Q. Let A_d be the set of all monic polynomials of degree exactly d in $\mathbb{F}_q[t]$. Obtain an estimate of the form

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