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Holonomic modules over Cherednik algebras, I



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ABSTRACT

The goal of this paper is to generalize several basic results from the theory of \mathcal{D} -modules to the representation theory of rational Cherednik algebras. We relate characterizations of holonomic modules in terms of singular support and Gelfand–Kirillov dimension. We study pullback, pushforward, and dual on the derived category of (holonomic) Cherednik modules for certain classes of maps between varieties. We prove, in the case of generic parameters for the rational Cherednik algebra, that pushforward with respect to an open affine inclusion preserves holonomicity.

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1. Introduction

The rational Cherednik algebra H_c of a finite subgroup W of $GL_n(\mathbb{C})$ is a universal deformation of the skew-product algebra $\mathcal{D}(\mathbb{C}^n) \rtimes CW$ of the Weyl algebra (algebra of polynomial differential operators on \mathbb{C}^n) with the group algebra of W (see [8, Theorem 2.23]). As such, it is a family of algebras over a space of parameters c , and in particular when we specialize the parameters to the numerical value $c = 0$ we just obtain

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$H_0 = \mathcal{D}(\mathbb{C}^n) \rtimes \mathbb{C}W$. Thus, we may view the representation theory of H_c as a deformation of the theory of W -equivariant \mathcal{D} -modules on affine space.

The foundational paper [12] studies a special category of representations of H_c which deforms the category of \mathcal{O} -coherent, graded W -equivariant \mathcal{D} -modules. For parameters c outside a countable collection of hyperplanes, this category is identical to the category of representations of W . In particular, the same is true of this category if $c = 0$.

Understanding the general representation theory of H_c remains a very difficult problem. I. Losev [15] has introduced the notion of *holonomic* representations of certain algebras, including rational Cherednik algebras, and it is the category of such representations that we study in the present paper. For generic parameters, we expect the theory of holonomic representations of H_c to be similar to the theory of W -equivariant holonomic \mathcal{D} -modules on \mathbb{C}^n (the $c = 0$ case). This expectation comes despite the fact that the Morita equivalence class of H_c may depend on c even for generic c : see [6] which classifies H_c up to Morita equivalence for W equal to the symmetric group. On the other hand, for special parameters, the presence of \mathcal{O} -coherent modules with less than full support as in [12] already shows that the category of holonomic Cherednik modules is more complicated than the theory of W -equivariant holonomic \mathcal{D} -modules.

More generally, one can study \mathcal{D} -modules on smooth varieties. P. Etingof [8] has defined a sheaf of Cherednik algebras on a smooth variety with the action of a finite group. The parameter space is more complicated (in particular, there is a global “twisting” parameter), but again by [8, Theorem 2.23] this sheaf of algebras is a universal deformation of the skew-product of the sheaf of differential operators on the variety with the group algebra.

The goal of this paper is to generalize several basic results from the theory of \mathcal{D} -modules to the current setting. The structure of the paper is as follows. After introducing the basic objects of study in section 2, we investigate in section 3 characterizations of holonomic modules in terms of singular support and Gelfand–Kirillov dimension. In section 4 we discuss the global setting of Cherednik modules on a variety, and we introduce pullback and pushforward for certain equivariant maps between varieties. We also introduce the Verdier dual functor on the derived category, and discuss an analog of Kashiwara’s theorem. In section 5, we prove, in the case of generic parameters for the rational Cherednik algebra, that pushforward with respect to open affine inclusions takes holonomic modules to holonomic modules over the rational Cherednik algebra.

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