Lie structure of truncated symmetric Poisson algebras

Ilana Z. Monteiro Alves, Victor Petrogradsky

PII: S0021-8693(17)30341-1
DOI: http://dx.doi.org/10.1016/j.jalgebra.2017.05.035
Reference: YJABR 16260

To appear in: *Journal of Algebra*

Received date: 23 December 2016

Please cite this article in press as: I.Z. Monteiro Alves, V. Petrogradsky, Lie structure of truncated symmetric Poisson algebras, *J. Algebra* (2017), http://dx.doi.org/10.1016/j.jalgebra.2017.05.035

LIE STRUCTURE OF TRUNCATED SYMMETRIC POISSON ALGEBRAS

ILANA Z. MONTEIRO ALVES AND VICTOR PETROGRADSKY

Abstract. The paper naturally continues series of works on identical relations of group rings, enveloping algebras, and other related algebraic structures. Let L be a Lie algebra over a field of characteristic $p > 0$. Consider its symmetric algebra $S(L) = \oplus_{n=0}^{\infty} U_n/U_{n-1}$, which is isomorphic to a polynomial ring. It also has a structure of a Poisson algebra, where the Lie product is traditionally denoted by $\{\ ,\ \}$. This bracket naturally induces the structure of a Poisson algebra on the ring $\mathbf{s}(L) = S(L)/(x^p \,|\, x \in L)$, which we call a truncated symmetric Poisson algebra. We study Lie identical relations of $\mathbf{s}(L)$. Namely, we determine necessary and sufficient conditions for L under which $\mathbf{s}(L)$ is Lie nilpotent, strongly Lie nilpotent, solvable and strongly solvable, where we assume that $p > 2$ to specify the solvability. We compute the strong Lie nilpotency class of $\mathbf{s}(L)$. Also, we prove that the Lie nilpotency class coincides with the strong Lie nilpotency class in case $p > 3$.

Shestakov proved that the symmetric algebra $S(L)$ of an arbitrary Lie algebra L satisfies the identity $\{x, \{y, z\}\} \equiv 0$ if, and only if, L is abelian. We extend this result for the (strong) Lie nilpotency and the (strong) solvability of $S(L)$. We show that the solvability of $\mathbf{s}(L)$ and $S(L)$ in case $\operatorname{char} K = 2$ is different from other characteristics, namely, we construct examples of such algebras which are solvable but not strongly solvable.

We use delta-sets for Lie algebras and the theory of identical relations of Poisson algebras. Also, we study filtrations in Poisson algebras and prove results on products of terms of the lower central series for Poisson algebras.

1. Introduction

The theory of associative PI-algebras, i.e. algebras satisfying nontrivial polynomial identities, is a classical area of the modern algebra [11]. This is an important instrument to study structure and properties of associative algebras. Now, there is an established theory of identical relations in Lie algebras [1]. It has many applications to group theory such as the solution of the Restricted Burnside Problem. Also, identical relations were applied to study other algebraic structures.

The first starting point for our research is the result of Passman on existence of identical relations in group rings [31] (Theorem 3.1). This paper caused an intensive research on different types of identical relations in group rings, such as Lie nilpotence, solvability, non-matrix identical relations, classes of Lie nilpotence, solvability lengths, etc. There are at least 50 papers published in this area.

Second, Latyshev [23] and Bahturin [2] started to study identical relations in universal enveloping algebras of Lie algebras. Passman [32] and Petrogradsky [33] specified existence of identical relations in restricted enveloping algebras (Theorem 3.4). There are many papers in this area studying different types of identical relations, such as Lie nilpotence, solvability, non-matrix identical relations, classes of Lie nilpotence, solvability lengths, etc. In particular, Riley and Shalev determined necessary and sufficient conditions for restricted Lie algebras under which the restricted enveloping algebra is Lie nilpotent or solvable [37]. The research was further extended to new objects, such as Lie superalgebras, color Lie superalgebras, smash products. These problems were studied in numerous papers by Bahturin, Bergen, Kochetov, Petrogradsky, Riley, Shalev, Siciliano, Spinelli, Usefi, et.al.

Poisson algebras appeared in works of Berezin [7] and Vergne [54]. Free Poisson algebras were introduced by Shestakov [40]. A basic theory of identical relations for Poisson algebras was developed by Farkas [13, 14]. Identical relations of symmetric Poisson algebras of Lie (super)algebras were studied by Kostant [21], Shestakov [40], and Farkas [14]. The third starting point for our research is the result of Giambruno and Petrogradsky [15] on existence of non-trivial multilinear Poisson identical relations in truncated symmetric algebras of restricted Lie algebras (Theorem 3.11).

2000 *Mathematics Subject Classification.* 17B63, 17B50, 17B01, 17B30, 17B65, 16R10.

Key words and phrases. Poisson algebras, identical relations, solvable Lie algebras, nilpotent Lie algebras, symmetric algebras, truncated symmetric algebras, restricted Lie algebras.

The second author was partially supported by grants FEMAT, CNPq 309542/2016-2

June 13, 2017.