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Group graded basic Morita equivalences

Tiberiu Coconeţ [a,*], Andrei Marcus [b]

[a] *Faculty of Economics and Business Administration, Babeş-Bolyai University, Str. Teodor Mihali, nr. 58-60, 400591 Cluj-Napoca, Romania*
[b] *Faculty of Mathematics, Babeş-Bolyai University, Cluj-Napoca, Romania*

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We introduce group graded basic Morita equivalences between algebras determined by blocks of normal subgroups, and by using the extended Brauer quotient, we show that they induce graded basic Morita equivalences at local levels.

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1. Introduction

Categorical equivalences between blocks of group algebras have been intensely studied over the last four decades, as they provide a structural explanation for various character correspondences which have been observed much earlier. It has turned out that in many

* Corresponding author.
E-mail addresses: tiberiu.coconet@math.ubbcluj.ro (T. Coconeţ), andrei.marcus@math.ubbcluj.ro (A. Marcus).

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cases, these are not mere equivalences between algebras – they are also compatible with the p-local structure of the blocks, encoded in terms of Brauer pairs or local pointed groups associated to subgroups of the defect groups, and their fusions. The motivation is related to the older idea of obtaining information about the block from information about blocks of local subgroups.

In the case of principal blocks, J. Rickard [17] introduced the so-called splendid derived equivalences, which involve permutation bimodules with diagonal vertices and trivial sources, with the feature that they give rise to derived equivalences between principal blocks of centralizers of p-subgroups. Splendid equivalences have been generalized to arbitrary blocks by M. Linckelmann [8,9] and M.E. Harris [5]. Later, a far reaching generalization has been achieved by L. Puig [13], who introduced basic equivalences, which also have a significant local structure, and induce equivalences between block of the centralizers of subgroups of the defect groups by employing the Brauer construction. Moreover, L. Puig and Y. Zhou [15,16] proved that these local equivalences extend to equivalences between blocks of the normalizers of subgroups of the defect groups. It was pointed out by X. Hu [6] that in fact, one gets in [15] graded Morita equivalences, via the construction introduced in [10].

In this paper, we investigate the local structure of graded Morita equivalences in a general setting. We fix a complete discrete valuation ring of characteristic zero with residue field k of characteristic $p > 0$, finite groups G and G' with normal subgroups N and N' respectively, such that the factor groups G/N and G'/N' are isomorphic. Let $\ddot{G}$ be the diagonal subgroup of $G \times G'$ with respect to the given isomorphism between G/N and G'/N'. Let b be a G-invariant block of $\mathscr{O}N$, and let b' be a G'-invariant block of $\mathscr{O}N'$.

A G/N-graded bimodule $\ddot{M}$ inducing a Morita equivalence between $A = \mathscr{O}Gb$ and $A' = \mathscr{O}G'b'$ has an 1-component which can be regarded as an indecomposable $\mathscr{O}\ddot{G}$-module, so it has a vertex $\ddot{P}$ and a source $\ddot{N}$. The discussion of the Morita equivalences in terms of $\ddot{P}$ and $\ddot{N}$ requires the introduction of a natural group graded structure on Puig's $\mathscr{O}G$-interior Hecke algebra $\mathrm{End}_{\mathscr{O}(1\times G')}(\mathrm{Ind}_{\ddot{P}}^{G\times G'}(\ddot{N}))$. We studied this graded structure in detail in [2]. In Section 3 we characterize in these terms the G/N-graded Morita equivalences between $\mathscr{O}Gb$ and $\mathscr{O}G'b'$, by linking, via the source module $\ddot{N}$, a defect pointed group P_γ of $G_{\{b\}}$ on $\mathscr{O}Nb$ and a defect pointed group $P'_{\gamma'}$ of $G'_{\{b'\}}$ on $\mathscr{O}N'b'$. Our first main result, Theorem 3.9 below, relates the Morita equivalence between the block extensions A and A' induced by $\ddot{M}$ to a G/N-graded Morita equivalence between the source algebras A_γ and $A'_{\gamma'}$.

In Section 4 we introduce the graded version of basic Morita equivalence between $\mathscr{O}Gb$ and $\mathscr{O}G'b'$, and we show in Corollary 4.3 that the truncation (that is, restriction of the grading to a subgroup of Γ) of a group graded basic Morita equivalence is again a group graded basic Morita equivalence.

The extended Brauer quotient of $\mathscr{O}G$ with respect to a p-subgroup Q of G was introduced in [15, Section 3], and it was generalized to N-interior G-algebras in [3]. In our situation, we show in Section 5 that the G/N-grading on $\mathscr{O}G$ induces a group grading

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