Accepted Manuscript

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PII: S0021-8693(17)30362-9
DOI: http://dx.doi.org/10.1016/j.jalgebra.2017.06.018
Reference: YJABR 16273

To appear in: *Journal of Algebra*

Received date: 30 December 2016

Please cite this article in press as: X. Hu et al., Blocks with Frobenius inertial quotients, *J. Algebra* (2017), http://dx.doi.org/10.1016/j.jalgebra.2017.06.018

Blocks with Frobenius inertial quotients

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Abstract In the paper, we prove that a perfect isometry between a block of a finite group and its Brauer correspondent can be extended to a perfect isometry between a block of a larger finite group and its Brauer correspondent, with suitable hypotheses. We also obtain a similar result for derived equivalences.

Keywords: Finite groups; blocks; perfect isometries; derived equivalences

1. Introduction

Throughout the paper, p is a prime number, $\mathcal{O}$ is a complete discrete valuation ring with an algebraically closed residue field F of characteristic p and with a fraction field $\mathcal{K}$ of characteristic 0. In order to discuss ordinary characters of finite groups, we additionally assume that $\mathcal{K}$ is 'big enough' for all finite groups in the paper. Let G be a finite group. A block b of G over $\mathcal{O}$ is a central primitive idempotent of the group algebra $\mathcal{O}G$. The block b is principal, if b identically acts on an $\mathcal{O}G$-module affording the trivial ordinary character of G. It is known that Sylow p-subgroups of G are defect groups of the principal block of G.

Let G be a finite group and b a block of G with defect group P. Assuming that P is abelian, Broué conjectures that the block b and its Brauer correspondent in $N_G(P)$ are derived equivalent (see [3, Conjecture 4.9]), where $N_G(P)$ denotes the normalizer of P in G. In [15, A2], Rouquier observed the algebraic structure of nilpotent blocks (see [11]), and then raised a question: if the hyperfocal subgroup Q (see [13]) of (P, f) is abelian, where (P, f) is a maximal Brauer pair of the block b of G, are the block b and its Brauer correspondent in $N_G(Q)$ basically Rickard equivalent (see [12])?

It is well known that derived equivalences between blocks imply perfect isometries (see [2]). In [17], Watanabe investigated the question at the level of characters of finite groups. She proved that if Q is contained in $Z(P)$ and is cyclic, then the principal blocks of G and $N_G(P)$ are isotypic (see [2]). She listed an example (see [17, Example 2]) to show that the condition $Q \leq Z(P)$ is necessary for the existence of the isotypy. In Watanabe's situation, the Brauer correspondents of the block b in $N_G(P)$ and of that in $N_G(Q)$ are basically Morita equivalent (see [17, Corollary 1]).

In the present paper, we investigate the question of Rouquier. Let G be a finite group and H a normal subgroup of G. Let b be a block of H stabilized by the G-conjugation. Then b is also a block of G. Let P be a defect group of the block b of G and set $Q = P \cap H$. Then Q is a defect group of the block of H. Let (Q, e) be a Brauer pair of the block b of H. Set $E_H(Q, e) = N_H(Q, e)/C_H(Q)$, where $C_H(Q)$ denotes the centralizer of Q in H. Assume that Q is contained in the center of P, that G is equal to PH, and that $E_H(Q, e)$ is cyclic and acts freely on the nontrivial elements of Q.

Theorem 1. *Keep the notation and the assumptions as above. Assume that Alperin's weight conjecture holds for the block b of H. Then the block b of G and its Brauer correspondent c in $N_G(P)$ are perfectly isometric.*