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One-dimensional Gorenstein local rings with decreasing Hilbert function

Anna Oneto, Francesco Strazzanti, Grazia Tamone

To the memory of Professor Anthony V. Geramita.

Abstract. In this paper we solve a problem posed by M.E. Rossi: *Is the Hilbert function of a Gorenstein local ring of dimension one not decreasing?* More precisely, for any integer $h > 1$, $h \notin \{14 + 22k,\, 35 + 46k \mid k \in \mathbb{N}\}$, we construct infinitely many one-dimensional Gorenstein local rings, included integral domains, reduced and non-reduced rings, whose Hilbert function decreases at level h; moreover, we prove that there are no bounds to the decrease of the Hilbert function. The key tools are numerical semigroup theory, especially some necessary conditions to obtain decreasing Hilbert functions found by the first and the third author, and a construction developed by V. Barucci, M. D'Anna and the second author, that gives a family of quotients of the Rees algebra. Many examples are included.

Introduction

Given a one-dimensional Cohen-Macaulay local ring $(R, \mathfrak{m}, k)$, let $G = \oplus_{h\geq 0}\left(\mathfrak{m}^h/\mathfrak{m}^{h+1}\right)$ be the associated graded ring and let H_R be the Hilbert function of R, defined as $H_R(h) = H_G(h) = \dim_k\left(\mathfrak{m}^h/\mathfrak{m}^{h+1}\right)$. The Cohen-Macaulayness of G and the behaviour of the Hilbert function are classic topics in local algebra. Starting from 1970s with the basic results of J.D. Sally [23], [24], [25], many authors have contributed to these themes; we recall J. Elias [13], M.E. Rossi and G. Valla [22] and Rossi's survey [21]. As is well-known, if G is Cohen-Macaulay the function H_R is non-decreasing. On the other hand, when $\operatorname{depth}(G) = 0$, H_R can decrease, i.e. $H_R(h-1) > H_R(h)$ for some h; in this case we say that H_R decreases at level h. When R is Gorenstein M.E. Rossi asked in [21, Problem 4.9] if H_R is always non-decreasing and in the last decade several authors found partial positive answers to this problem, especially in the case of numerical semigroup rings:

- in [2] F. Arslan and P. Mete for large families of complete intersection rings and the Gorenstein numerical semigroup rings with embedding dimension 4, under some arithmetical conditions
- in [3] F. Arslan, P. Mete and M. Şahin for infinitely many families of Gorenstein rings obtained by introducing the notion of nice gluing of numerical semigroups
- in [19] D.P. Patil and the third author for the rings associated with balanced numerical semigroups with embedding dimension 4
- in [4] F. Arslan, N. Sipahi and N. Şahin for other 4-generated Gorenstein numerical semigroup rings, constructed by non-nice gluing

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