Journal of Algebra 483 (2017) 329–361

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Isomorphisms between simple modules of degenerate cyclotomic Hecke algebras ☆

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ARTICLE INFO

Article history:
Received 17 November 2016
Available online 13 April 2017
Communicated by Leonard L. Scott, Jr.

Keywords:
Degenerate cyclotomic Hecke algebras
Generalized Mullineux involution
Categorifications
Schur–Weyl duality

ABSTRACT

We give explicit isomorphisms between simple modules of degenerate cyclotomic Hecke algebras defined via various cellular bases. A special case gives a generalized Mullineux involution in the degenerate case.

1. Introduction

Throughout, we work over the complex field $\mathbb{C}$. For any positive integer r, let $\mathscr{H}_r^{\mathrm{aff}}$ be the degenerate affine Hecke algebra. By definition, $\mathscr{H}_r^{\mathrm{aff}}$ is the unital associative $\mathbb{C}$-algebra generated by $s_1, \dots, s_{r-1}$ and $x_1, \dots, x_r$ which satisfy the following relations:

$$s_i^2 = 1, \text{ for } 1 \le i < r, \qquad x_i x_j = x_j x_i, \text{ if } 1 \le i, j \le r, \tag{1.1}$$

$$s_i s_j = s_j s_i, \text{ if } |i-j| > 1, \qquad s_i x_j = x_j s_i, \text{ if } j \ne i, i+1, \tag{1.2}$$

☆ Rui is supported by NSFC (grant No. 11571108). Song is supported by NSFC (grant No. 11501368).

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http://dx.doi.org/10.1016/j.jalgebra.2017.03.034

$$x_i s_i - s_i x_{i+1} = -1, \text{ if } 1 \le i < r, \qquad s_i x_i - x_{i+1} s_i = -1, \text{ if } 1 \le i < r, \tag{1.3}$$

$$s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1}, \text{ if } 1 \le i < r-1. \tag{1.4}$$

Let $\mathfrak{S}_r$ be the symmetric group on r letters. Then the group algebra $\mathbb{C}\mathfrak{S}_r$ is isomorphic to the subalgebra of $\mathscr{H}_r^{\text{aff}}$ generated by $\{s_i \mid 1 \le i \le r-1\}$. Moreover, the isomorphism sends the simple transposition $(i, i+1) \in \mathfrak{S}_r$ to s_i for all $1 \le i \le r-1$.

Let $\omega = (\omega_1, \ldots, \omega_\ell) \in \mathbb{C}^\ell$. The *degenerate cyclotomic Hecke algebra* or degenerate Hecke algebra of type $G(\ell, 1, r)$ is

$$\mathscr{H}_{\ell,r} = \mathscr{H}_r^{\text{aff}} / \langle f(x_1) \rangle, \tag{1.5}$$

where $\langle f(x_1) \rangle$ is the two-sided ideal of $\mathscr{H}_r^{\text{aff}}$ generated by

$$f(x_1) = (x_1 - \omega_1) \cdots (x_1 - \omega_\ell).$$

It is proved in [1] that $\mathscr{H}_{\ell,r}$ is a cellular algebra over the poset $\Lambda_{\ell,r}$ in the sense of [18], where $\Lambda_{\ell,r}$ is the set of ℓ-partitions of r. For each $\lambda \in \Lambda_{\ell,r}$, there is a cell module, say $S(\lambda)$, on which there is an invariant form ϕ_λ. Let $\text{rad}\phi_\lambda$ be the radical of ϕ_λ. By Graham–Lehrer's results on cellular algebras in [18], the quotient module $D(\lambda) := S(\lambda)/\text{rad}\phi_\lambda$ is either zero or simple and all non-zero $D(\lambda)$'s give a complete set of non-isomorphic simple $\mathscr{H}_{\ell,r}$-modules.

It is possible to show the existence of various cellular structures associated to $\mathscr{H}_{\ell,r}$. These structures, in turn, induce several ways to study the representation theory of the algebra. In particular, each of these structures depends on one dimensional representations of certain Young subgroups of $\mathfrak{S}_r$. For example, a cellular basis of $\mathscr{H}_{\ell,r}$ has been constructed via trivial representations of certain Young subgroups of $\mathfrak{S}_r$ in [1], whereas another one is defined via sign representations of certain Young subgroups of $\mathfrak{S}_r$ in non-degenerate case in [15, Remark 2.8].[1] Different structures really give different parameterizations of the simple modules of the algebra. It is quite natural to ask how these parameterizations are related.

Fix $\omega = (\omega_1, \ldots, \omega_\ell) \in \mathbb{C}^\ell$ and a 01-sequence $\underline{c} = (c_1, \ldots, c_\ell) \in \{0,1\}^\ell$. We construct various cellular bases of $\mathscr{H}_{\ell,r}$ in Corollary 2.2(a)–(b) via a class of one dimensional representations of certain Young subgroups of $\mathfrak{S}_r$. For any $\xi \in \mathfrak{S}_\ell$, define $\omega^\xi = (\omega_{(1)\xi}, \ldots, \omega_{(\ell)\xi})$. If we use the ω^ξ instead of the ω in Corollary 2.2(a)–(b) for the special case $\underline{c_0} := 0^\ell$, we will get another two kinds of cellular bases in Corollary 2.2(c)–(d). The corresponding simple $\mathscr{H}_{\ell,r}$-modules defined via the cellular bases in Corollary 2.2(a)–(d) are denoted by $D^{\underline{c}}(\lambda)$, $\tilde{D}^{\underline{c}}(\lambda)$, $D^\xi(\lambda)$ and $\tilde{D}^\xi(\lambda)$, respectively. The aim of this paper is to establish explicit isomorphisms between these simple modules.

[1] See [29] in degenerate case when $\ell = 2$.