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Cosilting complexes and AIR-cotilting modules ☆

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A B S T R A C T

We introduce and study the new concepts of cosilting complexes, cosilting modules and AIR-cotilting modules. We prove that the three concepts AIR-cotilting modules, cosilting modules and quasi-cotilting modules coincide with each other, in contrast with the dual fact that AIR-tilting modules, silting modules and quasi-tilting modules are different. Further, we show that there are bijections between the following four classes (1) equivalence classes of AIR-cotilting (resp., cosilting, quasi-cotilting) modules, (2) equivalence classes of 2-term cosilting complexes, (3) torsion-free cover classes and (4) torsion-free special precover classes. We also extend a classical result of Auslander and Reiten on the correspondence between certain contravariantly finite subcategories and cotilting modules to the case of cosilting complexes.

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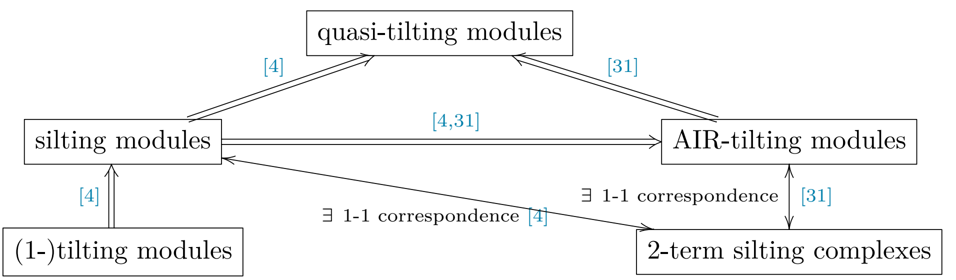
1. Introduction

The tilting theory is well known, and plays an important role in the representation theory of Artin algebra. The classical notion of tilting and cotilting modules was first considered in the case of finite dimensional algebras by Brenner and Butler [10] and by Happel and Ringel [18]. Cotilting theory (for arbitrary modules over arbitrary unital rings) extends Morita duality in analogy to the way tilting theory extends Morita equivalence. In particular, cotilting modules generalize injective cogenerators similarly as tilting modules generalize progenerators. Later, many scholars have done a lot of research on the tilting theory and cotilting theory, for instance [3,5–7,14,15,23,27,29] and so on.

In derived categories, silting complexes were first introduced by Keller and Vossieck [20] to study t-structures in the bounded derived category of representations of Dynkin quivers. Beginning with [2], such objects were recently shown to have various nice properties [21,22]. The results in [28] showed that silting complexes (i.e., semi-tilting complexes in [28]) have similar properties as that tilting modules have in the module categories. The recent two papers by Buan and Zhou [12,13] also showed that it is reasonable to see the silting theory as the relative version of tilting theory in the level of derived categories (while the tilting complexes [24] play the role of progenerators).

The τ-tilting theory recently introduced by Adachi, Iyama and Reiten [1] is an important generalization of the classical tilting theory. In particular, it was shown that support τ-tilting modules have close relations with 2-term silting complexes and cluster-tilting objects [1]. In [4], the authors introduced silting modules as a generalization of support τ-tilting modules over arbitrary rings and modules. We note that there is also another little different generalization of support τ-tilting modules over arbitrary rings which was firstly introduced by the second author [31], and was called AIR-tilting modules now (to distinguished with silting modules).

The following diagram gives the relationships of quasi-tilting modules, silting modules, AIR-tilting modules, (1-)tilting modules and 2-term silting complexes.



On the other hand, cotilting theory was also studied as the dual of tilting theory. Recall that an R-module T is *(n-)cotilting* (see for instance [7]) if it satisfies the following three conditions (1) id$T \leqslant n$, i.e., the injective dimension of T is finite, (2) $\mathrm{Ext}_R^i(T^X, T) = 0$ for any X, and (3) there is an exact sequence $0 \to T_n \to \cdots \to T_0 \to Q \to 0$, where