



Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra



Fine gradings on simple exceptional Jordan pairs and triple systems



Diego Aranda-Orna¹

Departamento de Matemáticas e Instituto Universitario de Matemáticas y Aplicaciones, Universidad de Zaragoza, 50009 Zaragoza, Spain

ARTICLE INFO

Article history:

Received 21 August 2015

Available online 30 August 2017

Communicated by Nicolás Andruskiewitsch

Keywords:

Fine gradings

Exceptional Jordan pairs

Lie algebras

Exceptional Jordan triple systems

ABSTRACT

We give a classification up to equivalence of the fine group gradings by abelian groups on the Jordan pairs and triple systems of types bi-Cayley and Albert, under the assumption that the base field is algebraically closed of characteristic different from 2. The associated Weyl groups are computed. We also determine, for each fine grading on the bi-Cayley and Albert pairs, the induced grading on the exceptional simple Lie algebra given by the Tits–Kantor–Koecher construction.

© 2017 Elsevier Inc. All rights reserved.

1. Introduction and preliminaries

In the classification of simple Jordan pairs ([16]) there are four infinite families and two exceptional cases. The importance of Jordan pairs is due to the Tits–Kantor–Koecher (TKK) construction, which allows to construct a 3-graded Lie algebra from a Jordan pair, which is simple if and only if so is the Jordan pair. It is also known that a 3-graded

E-mail address: daranda@unizar.es.

¹ Supported by the Spanish Ministerio de Economía y Competitividad—Fondo Europeo de Desarrollo Regional (FEDER) MTM2013-45588-C3-2-P and by the Diputación General de Aragón—Fondo Social Europeo (Grupo de Investigación de Álgebra).

Lie algebra determines a Jordan pair. The exceptional simple Jordan pairs, namely, the types bi-Cayley and Albert, allow to construct Lie algebras of types E_6 and E_7 via the TKK construction.

The main goal of this paper is to study and classify fine gradings on Jordan pairs and triple systems of types bi-Cayley and Albert. By grading we usually mean grading by an abelian group, and we always assume that the base field $\mathbb{F}$ is algebraically closed of characteristic different from 2, unless otherwise stated.

This paper is organized as follows.

In this section, we first give the basic definitions relevant to Jordan pairs and triple systems, and then recall the classification of orbits under the action of the automorphism group of finite-dimensional simple Jordan pairs. Finally, we recall the classification of fine group gradings on the Cayley algebra and on the Albert algebra, which will be used later to construct gradings on the Jordan systems under consideration. A nice reference to study the classification of fine gradings on Cayley and Albert algebras is [9].

In the second section, we introduce the basic definitions concerning gradings on Jordan pairs and triple systems, and prove some general results about such gradings. In particular, we study how the gradings on a Jordan pair induce gradings on the associated Lie algebra given by the TKK construction. We prove, using Peirce decompositions, that fine gradings on finite-dimensional semisimple Jordan pairs have 1-dimensional homogeneous components. We also prove that the trace form behaves well with respect to the gradings on Jordan pairs and triple systems.

In the third section we recall the definition of the bi-Cayley and Albert pairs and triple systems, and introduce some important automorphisms. For the bi-Cayley systems, we describe the automorphism groups and their orbits. A construction of fine gradings on the bi-Cayley and Albert pairs and triple systems is given.

The main goal of the paper is reached in the fourth section, where we classify, up to equivalence, the fine gradings by abelian groups on the bi-Cayley and Albert pairs and triple systems (over an algebraically closed field of characteristic not 2).

In the fifth section, we study how the fine gradings on the bi-Cayley and Albert pairs induce gradings on the Lie algebra obtained by means of the TKK construction.

Finally, in the sixth section we compute the Weyl groups of all the fine gradings on the bi-Cayley and Albert pairs and triple systems.

1.1. Jordan pairs and triple systems

We will now recall from [16] some basic definitions about Jordan pairs and Jordan triple systems.

As already mentioned, we will assume throughout the paper that the ground field $\mathbb{F}$ is algebraically closed of characteristic different from 2, unless indicated otherwise. The superscript σ will always take the values $+$ and $-$, and will be omitted when there is no ambiguity.

Download English Version:

<https://daneshyari.com/en/article/5771818>

Download Persian Version:

<https://daneshyari.com/article/5771818>

[Daneshyari.com](https://daneshyari.com)