Journal of Algebra 477 (2017) 1–28

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

The based ring of the lowest generalized two-sided cell of an extended affine Weyl group ☆

Xun Xie [a,b,*]

[a] *Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China*
[b] *School of Mathematical Sciences, Peking University, Beijing 100871, China*

A R T I C L E   I N F O

Article history:
Received 20 September 2015
Available online 3 January 2017
Communicated by Gunter Malle

MSC:
20C08
20G05

Keywords:
Affine Hecke algebra
Lowest two-sided cell
Based ring
P1–P15

A B S T R A C T

Let $\mathbf{c}_0$ be the lowest generalized two-sided cell of an extended affine Weyl group W with unequal parameters. We first prove that certain conjectures of Lusztig (called P1–P15) hold for $\mathbf{c}_0$, which implies the existence of the based ring of $\mathbf{c}_0$, and then we determine the structure of the based ring. As an application, we use the structure of the based ring to study certain simple modules of the Hecke algebra with unequal parameters associated to W, namely those attached to $\mathbf{c}_0$. Further we give a set of prime ideals $\mathfrak{p}$ of the center $\mathcal{Z}$ of the affine Hecke algebra $\mathcal{H}$ such that the reduced algebra $k_\mathfrak{p}\mathcal{H}$ is simple over $k_\mathfrak{p}$, where $k_\mathfrak{p} = \mathrm{Frac}(\mathcal{Z}/\mathfrak{p})$ is the residue field of $\mathcal{Z}$ at $\mathfrak{p}$. In particular, we show that the algebra $\mathcal{H} \otimes_\mathcal{Z} \mathrm{Frac}(\mathcal{Z})$ is split simple over the field $\mathrm{Frac}(\mathcal{Z})$.

☆ Partially supported by the National Natural Science Foundation of China (No. 11321101).

* Correspondence to: Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China.

E-mail address: xieg7@163.com.

http://dx.doi.org/10.1016/j.jalgebra.2016.11.031
0021-8693/© 2016 Elsevier Inc. All rights reserved.

1. Introduction

Two-sided cells of Coxeter groups were introduced in [8]. They play an important role in the representation theory of Hecke algebras. Lusztig extensively studied the two-sided cells of affine Weyl groups in [12–15]. In [13], he introduced the concept of the based ring (also called the asymptotic Hecke algebra), which provides an algebraic way to study representations of Hecke algebras; see for examples [14,15,19,21,22].

The two-sided cells can also be defined for Coxeter groups with unequal parameters; see [16]. There is also the concept of based ring, but we need to assume some conjectural properties, which are called P1–P15 (see [16, §14]).

In this paper, we are concerned with the based ring of the lowest two-sided cell of an extended affine Weyl group W with unequal parameters. We prove P1–P15 for the lowest two-sided cell, which implies the existence of the based ring of the lowest two-sided cell. We then determine the structure of the based ring. As an application, we use these results to study certain representations of affine Hecke algebras with unequal parameters.

The content of this paper is as follows. In §2 we recall some basic facts about extended affine Weyl groups, their Hecke algebras and the lowest two-sided cells. In §3 we study a formula which is due to Xi in the equal parameter case. This formula gives a decomposition of the Kazhdan–Lusztig element C_w for w in the lowest two-sided cell. This formula is crucial for determining the structure of the based ring of the lowest two-sided cell. In §4 we show that the properties P1–P15 are valid for the lowest two-sided cell. In §5 we determine the based ring of the lowest two-sided cell. In §6 we consider the based ring of the lowest two-sided cell of an affine Weyl group of type $\tilde{C}_n$. In §7 we use the ideas in [21,22] to study simple representations of an affine Hecke algebras with unequal parameters attached to the lowest two-sided cell. In §8 we study the reduced affine Hecke algebra $k_\mathfrak{p}\mathcal{H} = \mathcal{H} \otimes_\mathcal{Z} k_\mathfrak{p}$, where $\mathcal{H}$ is the generic affine Hecke algebra, $\mathcal{Z}$ is the center of $\mathcal{H}$, $\mathfrak{p}$ is a prime ideal of $\mathcal{Z}$, and $k_\mathfrak{p} = \mathrm{Frac}(\mathcal{Z}/\mathfrak{p})$ is the residue field of $\mathcal{Z}$ at $\mathfrak{p}$.

2. Preliminaries

In this section, we recall some basic definitions and facts about affine Weyl groups, their Hecke algebras and the lowest two-sided cells.

2.1. Extended affine Weyl groups

Let R be an irreducible root system. Its Weyl group, root lattice and weight lattice are denoted by W_0, Q and P respectively. Then $W' = W_0 \ltimes Q$ is an affine Weyl group and $W = W_0 \ltimes P = \Omega \ltimes W'$ is an extended affine Weyl group, where Ω is a finite subgroup of W and is isomorphic to P/Q. The length function $l : W' \to \mathbb{N}$ can be extended to a function $l : W \to \mathbb{N}$ by setting $l(\omega w) = l(w)$ for $\omega \in \Omega$ and $w \in W'$. We always denote by e the neutral element of the group W, and by w_0 the longest element of W_0.