Gorenstein and Totally Reflexive Orders

Josh Stangle

PII: S0021-8693(16)30496-3
DOI: http://dx.doi.org/10.1016/j.jalgebra.2016.12.021
Reference: YJABR 16041

To appear in: *Journal of Algebra*

Received date: 26 April 2016

GORENSTEIN AND TOTALLY REFLEXIVE ORDERS

JOSH STANGLE

ABSTRACT. In this paper we study orders over Cohen-Macaulay rings. We discuss the properties needed for these orders to give noncommutative crepant resolutions of the base rings; namely, we want algebraic analogs of birationality, nonsingularity, and crepancy. While some definitions have been made, we discuss an alternate definition and obstructions to the existence of such objects. We then give necessary and sufficient conditions for an order to have certain desirable homological properties. We examine examples of rings satisfying these properties to prove that certain endomorphism rings over abelian quotient singularities have infinite global dimension.

1. INTRODUCTION

In [13], Van den Bergh introduced the notion of a *noncommutative (crepant) resolution* of a commutative ring R as an algebraic analog of desingularizations in algebraic geometry. This is a certain endomorphism ring, $\mathrm{End}_R(M)$, over a commutative ring R, which is also a maximal Cohen-Macaulay R-module and has finite global dimension. A great deal of work has been done in the time since. In the case where R is a Gorenstein ring, Van den Bergh's definition leads to strong theorems and connections to geometric notions. For example, appropriate conditions on R guarantee that R having a non-commutative crepant resolution is equivalent to $\mathrm{Spec} R$ having a commutative crepant resolution. This is the material of Section 2, where some work of Van den Bergh and Stafford-Van den Bergh [12, 13] and Leuschke and Buchweitz-Leuschke-Van den Bergh [2, 10] is presented.

In the case where R is not Gorenstein, the situation is much less clear. One goal of study is trying to find a construction in this case which will give some of the main results from the Gorenstein case, in particular Theorems 2.3, 2.5, and 2.7. Work in this setting has been done in large part by Dao, Ingalls, and Faber in [5]; Dao, Iyama, Takahashi, and Vial in [6]; and Iyama and Wemyss in [9]. In these articles, the definition of a noncommutative crepant resolution in the Gorenstein case is applied to the non-Gorenstein case. This leads to some positive results, but the loss of some strong theorems, as in Example 3.3.

Date: December 29, 2016.

Keywords. Noncommutative crepant resolutions, Gorenstein orders, Endormorphism Rings