Journal of Algebra 477 (2017) 163–194

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

The representation dimension of a selfinjective algebra of wild tilted type

Ibrahim Assem [a], Andrzej Skowroński [b], Sonia Trepode [c,*]

[a] *Département de mathématiques, Faculté des sciences, Université de Sherbrooke, Sherbrooke, Québec J1K 2R1, Canada*
[b] *Faculty of Mathematics and Computer Science, Nicolaus Copernicus University, Chopina 12/18, 87-100 Toruń, Poland*
[c] *Departamento de Matemática, Facultad de Ciencias Exactas y Naturales, Funes 3350, Universidad Nacional de Mar del Plata, Conicet, UNMDP, 7600 Mar del Plata, Argentina*

ARTICLE INFO

Article history:
Received 1 July 2016
Available online 2 January 2017
Communicated by Nicolás Andruskiewitsch

MSC:
16G20
16G60
16G70
18G20

Keywords:
Representation dimension
Selfinjective algebras
Tilted algebras
Wild type
Wild algebras

ABSTRACT

We prove that the representation dimension of a selfinjective algebra of wild tilted type is equal to three, and give an explicit construction of an Auslander generator of its module category. We also show that if a connected selfinjective algebra admits an acyclic generalised standard Auslander–Reiten component then its representation dimension is equal to three.

* Corresponding author.
E-mail address: strepode@gmail.com (S. Trepode).

http://dx.doi.org/10.1016/j.jalgebra.2016.12.010

1. Introduction

Our objective in this paper is to explore the relation between the representation theory of an algebra, or more precisely the shape of its Auslander–Reiten components, and its homological invariants. We are in particular interested here in the representation dimension of an algebra, introduced by Auslander in [8], which measures in some way the complexity of the morphisms of the module category. There were several attempts to understand, or compute, this invariant, see, for instance, [8,15,3]. Special attention was given to algebras of representation dimension three. The reason for this interest is two-fold. Firstly, it is related to the finitistic dimension conjecture: Igusa and Todorov have proved that algebras of representation dimension three have a finite finitistic dimension [19]. Secondly, because Auslander's expectation was that the representation dimension would measure how far an algebra is from being representation-finite, there is a standing conjecture that the representation dimension of a tame algebra is at most three. Indeed, while there exist algebras of arbitrary [28], but finite [20], representation dimension, most of the best understood classes of algebras have representation dimension at most three. This is the case, for instance, for algebras obtained by means of tilting, such as tilted algebras [5], iterated tilted algebras [11] and quasitilted algebras [25]. This is also the case for classes of selfinjective algebras related to the ones obtained via tilting, such as trivial extensions of iterated tilted algebras [12] and selfinjective algebras of euclidean type [7]. In both of these cases, the algebra considered is the orbit algebra of the repetitive algebra of some tilted algebra under the action of an infinite cyclic group of automorphisms.

It was then natural to consider next the class of selfinjective algebras of wild tilted type, introduced and studied in [16]. A selfinjective algebra A is called of *wild tilted type* if A is the orbit category of the repetitive category $\hat{B}$, in the sense of [18], of a tilted algebra B of wild type, under the action of an infinite cyclic group of automorphisms. Our first main theorem may now be stated.

Theorem A. *Let A be a connected selfinjective algebra of wild tilted type. Then* $\operatorname{rep.dim.} A = 3$.

Because the definition of our class is similar to that of the one considered in [7], we are able to follow the same general strategy of proof as in that paper. In particular, our proof is constructive and we are able to explicitly describe an Auslander generator of the module category of A. However, because we are dealing with wild algebras, the necessary constructions are different.

Returning to our basic problem of relating the shape of Auslander–Reiten components to the representation dimension, we are led to consider the case of selfinjective algebras having an acyclic generalised standard component. We recall than an Auslander–Reiten component Γ is called *generalised standard* [31] whenever, for two modules X, Y in Γ, we have $\operatorname{rad}_A^{\infty}(X, Y) = 0$, so that morphisms can be computed locally in that component.