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The recognition problem for table algebras and reality-based algebras

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Abstract

Given a finite-dimensional noncommutative semisimple algebra A over $\mathbb{C}$ with involution, we show that A always has a basis $\mathbf{B}$ for which $(A, \mathbf{B})$ is a reality-based algebra. For algebras that have a one-dimensional representation δ, we show that there always exists an RBA-basis for which δ is a positive degree map. We characterize all RBA-bases of the 5-dimensional noncommutative semisimple algebra for which the algebra has a positive degree map, and give examples of RBA-bases of $\mathbb{C} \oplus M_n(\mathbb{C})$ for which the RBA has a positive degree map, for all $n \geq 2$.

Key words : Table algebras, C-algebras, Reality-based algebras.
AMS Classification: Primary: 05E30; Secondary: 20C15.

1 Introduction

Let A be a $(d+1)$-dimensional involutive algebra over $\mathbb{C}$, whose involution $*$ is a ring antiautomorphism that restricts to complex conjugation on scalars. We say that the pair $(A, \mathbf{B})$ is a *reality-based algebra* (or RBA) if there is a basis $\mathbf{B} = \{b_0, b_1, \ldots, b_d\}$ of A such that

(i) the multiplicative identity of A is an element of $\mathbf{B}$ (we index the elements of $\mathbf{B}$ so that b_0 is the multiplicative identity of A);

(ii) $\mathbf{B}^2 \subseteq \mathbb{R}\mathbf{B}$, in particular the structure constants λ_{ijk} generated by the basis $\mathbf{B}$ in the expressions $b_i b_j = \sum_{k=0}^{d} \lambda_{ijk} b_k$ are all real numbers;

(iii) $\mathbf{B}^* = \mathbf{B}$, so $*$ induces a product of disjoint transpositions on the set $\{0, 1, \ldots, d\}$ given by $b_{i^*} = (b_i)^*$ for all $b_i \in \mathbf{B}$;

(iv) $\lambda_{ij0} \neq 0 \iff j = i^*$; and

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