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ABSTRACT. In 2006 Sommers and Tymoczko defined so called arrangements of ideal type $\mathscr{A}_{\mathcal{I}}$ stemming from ideals $\mathcal{I}$ in the set of positive roots of a reduced root system. They showed in a case by case argument that $\mathscr{A}_{\mathcal{I}}$ is free if the root system is of classical type or G_2 and conjectured that this is also the case for all types. This was established only very recently in a uniform manner by Abe, Barakat, Cuntz, Hoge and Terao. The set of non-zero exponents of the free arrangement $\mathscr{A}_{\mathcal{I}}$ is given by the dual of the height partition of the roots in the complement of $\mathcal{I}$ in the set of positive roots, generalizing the Shapiro-Steinberg-Kostant theorem which asserts that the dual of the height partition of the set of positive roots gives the exponents of the associated Weyl group.

Our first aim in this paper is to investigate a stronger freeness property of the $\mathscr{A}_{\mathcal{I}}$. We show that all $\mathscr{A}_{\mathcal{I}}$ are inductively free, with the possible exception of some cases in type E_8.

In the same paper from 2006, Sommers and Tymoczko define a Poincaré polynomial $\mathcal{I}(t)$ associated with each ideal $\mathcal{I}$ which generalizes the Poincaré polynomial $W(t)$ for the underlying Weyl group W. Solomon showed that $W(t)$ satisfies a product decomposition depending on the exponents of W for any Coxeter group W. Sommers and Tymoczko showed in a case by case analysis in type A_n, B_n and C_n, and some small rank exceptional types that a similar factorization property holds for the Poincaré polynomials $\mathcal{I}(t)$ generalizing the formula of Solomon for $W(t)$. They conjectured that their multiplicative formula for $\mathcal{I}(t)$ holds in all types. In our second aim to investigate this conjecture further, the same inductive tools we develop to obtain inductive freeness of the $\mathscr{A}_{\mathcal{I}}$ are also employed. Here we also show that this conjecture holds inductively in almost all instances with only a small number of possible exceptions.

CONTENTS

1. INTRODUCTION

Much of the motivation for the study of arrangements of hyperplanes comes from Coxeter arrangements. They consist of the reflecting hyperplanes associated with the reflections

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