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Twisted filtrations of Soergel bimodules and linear Rouquier complexes

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ABSTRACT

We consider twisted standard filtrations of Soergel bimodules associated to arbitrary Coxeter groups and show that the graded multiplicities in these filtrations can be interpreted as structure constants in the Hecke algebra. This corresponds to the positivity of the polynomials occurring when expressing an element of the canonical basis in a generalized standard basis twisted by a biclosed set of roots in the sense of Dyer, and comes as a corollary of Soergel's conjecture. We then show the positivity of the corresponding inverse polynomials in the case where the biclosed set is an inversion set of an element or its complement by generalizing a result of Elias and Williamson on the linearity of the Rouquier complexes associated to lifts of these basis elements in the Artin–Tits group. These lifts turn out to be generalizations of Mikado braids as introduced in a joint work with Digne. This second positivity property generalizes a result of Dyer and Lehrer from finite to arbitrary Coxeter groups.

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1. Introduction

Let $(\mathcal{W}, \mathcal{S})$ be a Coxeter system with $|\mathcal{S}| < \infty$. Let $\mathcal{H}$ be the corresponding Iwahori–Hecke algebra over the ring $\mathbb{Z}[v, v^{-1}]$ with standard basis $\{T_w\}_{w \in \mathcal{W}}$ and costandard basis $\{T_{w^{-1}}^{-1}\}_{w \in \mathcal{W}}$. Denote by $\mathcal{T}$ the set of conjugates of the elements of $\mathcal{S}$. In their seminal paper of 1979 [19], Kazhdan and Lusztig introduced two canonical bases $\{C_w\}_{w \in \mathcal{W}}$ and $\{C'_w\}_{w \in \mathcal{W}}$ of $\mathcal{H}$ and related them to the representation theory of $\mathcal{H}$ and $\mathcal{W}$. In the case where $\mathcal{W}$ is a finite Weyl group, the canonical bases are closely related to the geometry of Schubert varieties. Kazhdan and Lusztig conjectured that the coefficients of C'_w expressed in the standard basis are polynomials with nonnegative coefficients. These polynomials became known as *Kazhdan–Lusztig polynomials* and are broadly studied in Lie theory, representation theory and combinatorics (see for instance [2] or [17] for introductions to the topic).

While Kazhdan and Lusztig proved their positivity conjecture in 1980 in the case where $\mathcal{W}$ is a finite or affine Weyl group [20] using geometric techniques, the general case remained mysterious until recently. Soergel proposed [28,29] an approach allowing one to replace the geometry involved in the Weyl group case by a remarkable additive monoidal Krull–Schmidt category $\mathcal{B}$ of graded bimodules over a polynomial ring. These bimodules, nowadays called *Soergel bimodules*, can be defined for an arbitrary Coxeter system and provide a categorification of (the canonical basis $\{C'_w\}_{w \in \mathcal{W}}$ of) $\mathcal{H}$. In this framework, Soergel formulated a purely algebraic conjecture implying Kazhdan–Lusztig's positivity conjecture in full generality [29] and proved it for finite Weyl groups, using again geometry but suggesting the existence of an algebraic proof. Soergel's conjecture was proven by Elias and Williamson in [14].

More precisely, indecomposable Soergel bimodules are indexed (up to graduation shifts and isomorphism) by the elements of $\mathcal{W}$. Denote by $\{B_w\}_{w \in \mathcal{W}}$ the family of (unshifted)