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The max-plus algebra of exponent matrices of tiled orders

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Abstract

An exponent matrix is an $n \times n$ matrix $A = (a_{ij})$ over \mathbb{N}^0 satisfying (1) $a_{ii} = 0$ for all $i = 1, \dots, n$ and (2) $a_{ij} + a_{jk} \geq a_{ik}$ for all pairwise distinct $i, j, k \in \{1, \dots, n\}$. In the present paper we study the set \mathcal{E}_n of all non-negative $n \times n$ exponent matrices as an algebra with the operations \oplus of component-wise maximum and \odot of component-wise addition. We provide a basis of the algebra $(\mathcal{E}_n, \oplus, \odot, 0)$ and give a row and a column decompositions of a matrix $A \in \mathcal{E}_n$ with respect to this basis. This structure result determines all $n \times n$ -tiled orders over a fixed discrete valuation domain. We also study automorphisms of \mathcal{E}_n with respect to each of the operations \oplus and \odot and prove that $\text{Aut}(\mathcal{E}_n, \oplus, \odot, 0) \cong \text{Aut}(\mathcal{E}_n, \oplus) \cong \text{Aut}(\mathcal{E}_n, \odot) \cong \mathcal{S}_n \times C_2$, $n > 2$.

Keywords: Exponent matrix, max-plus algebra, tiled order

2010 MSC: 16H99, 16Z99, 15A80

1. Introduction

Orders over domains is a classical object of study, originated by Dedekind's ideal theory of maximal orders in algebraic number fields. Apart from their own interest as a “noncommutative arithmetic”, orders have also great importance to the theory of integral representations and to integer matrices [31]. Orders of tiled form appeared as structural ingredients in the study of hereditary orders [4], [20] (see also [31] and [33]), Bass orders [11] and, more generally, they are used in the context of quasi-Bass orders in [10]. The latter two references witness

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