Accepted Manuscript

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PII: S0021-8693(17)30380-0
DOI: http://dx.doi.org/10.1016/j.jalgebra.2017.05.045
Reference: YJABR 16291

To appear in: *Journal of Algebra*

Received date: 30 December 2016

The max-plus algebra of exponent matrices of tiled orders

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Abstract

An exponent matrix is an $n\times n$ matrix $A=(a_{ij})$ over $\mathbb{N}^0$ satisfying (1) $a_{ii}=0$ for all $i=1,\dots,n$ and (2) $a_{ij}+a_{jk}\geq a_{ik}$ for all pairwise distinct $i,j,k\in\{1,\dots,n\}$. In the present paper we study the set $\mathcal{E}_n$ of all non-negative $n\times n$ exponent matrices as an algebra with the operations $\oplus$ of component-wise maximum and $\odot$ of component-wise addition. We provide a basis of the algebra $(\mathcal{E}_n,\oplus,\odot,0)$ and give a row and a column decompositions of a matrix $A\in\mathcal{E}_n$ with respect to this basis. This structure result determines all $n\times n$-tiled orders over a fixed discrete valuation domain. We also study automorphisms of $\mathcal{E}_n$ with respect to each of the operations $\oplus$ and $\odot$ and prove that $\mathrm{Aut}(\mathcal{E}_n,\oplus,\odot,0)\cong\mathrm{Aut}(\mathcal{E}_n,\oplus)\cong\mathrm{Aut}(\mathcal{E}_n,\odot)\cong\mathcal{S}_n\times C_2$, $n>2$.

Keywords: Exponent matrix, max-plus algebra, tiled order
2010 MSC: 16H99, 16Z99, 15A80

1. Introduction

Orders over domains is a classical object of study, originated by Dedekind's ideal theory of maximal orders in algebraic number fields. Apart from their own interest as a "noncommutative arithmetic", orders have also great importance to the theory of integral representations and to integer matrices [31]. Orders of tiled form appeared as structural ingredients in the study of hereditary orders [4], [20] (see also [31] and [33]), Bass orders [11] and, more generally, they are used in the context of quasi-Bass orders in [10]. The latter two references witness

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