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THE FUNDAMENTAL THEOREM FOR WEAK BRAIDED BIMONADS

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Abstract. The theories of (Hopf) bialgebras and weak (Hopf) bialgebras have been introduced for vector space categories over fields and make heavily use of the tensor product. As first generalisations, these notions were formulated for monoidal categories, with braidings if needed. The present authors developed a theory of bimonads and Hopf monads H on arbitrary categories $\mathbb{A}$, employing distributive laws, allowing for a general form of the Fundamental Theorem for Hopf algebras. For τ-bimonads H, properties of braided (Hopf) bialgebras were captured by requiring a Yang-Baxter operator $\tau : HH \to HH$. The purpose of this paper is to extend the features of weak (Hopf) bialgebras to this general setting including an appropriate form of the Fundamental Theorem. This subsumes the theory of braided Hopf algebras (based on weak Yang-Baxter operators) as considered by Alonso Álvarez and others.

Contents

Introduction

There are various generalisations of the notions of (weak) bialgebras and Hopf algebras in the literature, mainly for (braided) monoidal categories, and we refer to Böhm [7], the introductions to Alonso Álvarez e.a. [2], Böhm e.a. [10], and [14, Remarks 36.18] for more information about these.

Bimonads and Hopf monads on *arbitrary* categories were introduced in [24] and the purpose of the present paper is to develop a *weak version* of these notion, that is, the initial conditions on the behaviour of the involved distributive laws towards unit and counit are replaced by weaker conditions.

Recall that for a bialgebra $(H, m, e, \delta, \varepsilon)$ over a commutative ring k, there is a commutative diagram ($\otimes_k = \otimes$)

$$\begin{array}{ccc} \mathbb{M} & \xrightarrow{-\otimes H} & \mathbb{M}_H^H \\ & {\scriptstyle \phi_H} \searrow \quad \swarrow {\scriptstyle U^{\widetilde{H}}} & \\ & \mathbb{M}_H & \end{array}, \qquad \begin{array}{ccc} M & \longmapsto & (M \otimes H, M \otimes m, M \otimes \delta) \\ & \searrow & \downarrow \\ & & (M \otimes H, M \otimes m)\,, \end{array}$$

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