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On a conjecture of Dao–Kurano

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ABSTRACT

We prove a special case of a conjecture of Dao–Kurano concerning the vanishing of Hochster's theta pairing. The proof uses Adams operations on both topological K-theory and perfect complexes with support.

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1. Introduction

Let A be a local hypersurface ring with maximal ideal η. Assume A has an isolated singularity; that is, assume $A_{\mathfrak{p}}$ is a regular local ring for all $\mathfrak{p} \in \operatorname{Spec}(A) \setminus \{\eta\}$. If M and N are finitely generated A-modules, $l(\operatorname{Tor}_i^A(M,N)) < \infty$ for $i \gg 0$, where $l(-)$ denotes length as an A-module. Further, since minimal free resolutions of finitely generated A-modules are eventually 2-periodic, $\operatorname{Tor}_i^A(M,N) = \operatorname{Tor}_{i+2}^A(M,N)$ for $i \gg 0$. This motivates the following definition:

Definition 1.1. Let M and N be finitely generated A-modules. The *Hochster theta pairing* applied to M and N is given by

$$\theta(M,N) = l(\operatorname{Tor}_{2i}^A(M,N)) - l(\operatorname{Tor}_{2i+1}^A(M,N)),\ i \gg 0.$$

The Hochster theta pairing was introduced by Hochster in [12]. Various conjectures concerning the vanishing of θ have received a good deal of attention lately: see, for instance, work of Buchweitz–van Straten ([5]), Dao ([6]), Moore–Piepmeyer–Spiroff–Walker ([15]), and Polishchuk–Vaintrob ([17]). For a more detailed history of the Hochster theta pairing, we refer the reader to Section 3 of Dao–Kurano's article [8].

Conjecture 3.1 of [8] lists several open questions regarding the vanishing of θ. In particular, Dao–Kurano conjecture the following:

Conjecture 1.2 *(Dao–Kurano, [8] Conjecture 3.1 (3)). Let A be a local hypersurface of Krull dimension d with isolated singularity, and let M and N be finitely generated A-modules. If* $\dim(M) \leqslant \frac{d}{2}$, $\theta(M,N) = 0$.

Dao–Kurano themselves prove Conjecture 1.2 in the following cases:

- $A = S_\eta$, where S is a positively graded algebra over a field k such that $\operatorname{Proj}(S)$ is smooth over k, and η is the homogeneous maximal ideal of S
- A is excellent, A contains a field of characteristic 0, and $d \leqslant 6$

The goal of this paper is to prove Conjecture 1.2 in the following additional special case:

Theorem 1.3. *Let $Q := \mathbb{C}[x_1, \dots, x_n]$, let $\mathfrak{m}$ denote the maximal ideal $(x_1, \dots, x_n) \subseteq Q$, let $f \in \mathfrak{m}$, and let $R := Q/(f)$. Assume the local hypersurface $R_{\mathfrak{m}}$ has an isolated singularity. Let M and N be finitely generated $R_{\mathfrak{m}}$-modules. If* $\dim(M) \leqslant \frac{n-1}{2}$, $\theta(M,N) = 0$.

Remark 1.4. When n is odd, Theorem 1.3 follows immediately from a theorem of Buchweitz–van Straten which implies that $\theta(M,N) = 0$ for *all* finitely generated $R_{\mathfrak{m}}$-modules M, N (see the Main Theorem on page 245 of [5]).