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Jordan Trialgebras and Post-Jordan Algebras

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JORDAN TRIALGEBRAS AND POST-JORDAN ALGEBRAS

FATEMEH BAGHERZADEH, MURRAY BREMNER, AND SARA MADARIAGA

ABSTRACT. We compute minimal sets of generators for the S_n-modules ($n \leq 4$) of multilinear polynomial identities of arity n satisfied by the Jordan product and the Jordan diproduct (resp. pre-Jordan product) in every triassociative (resp. tridendriform) algebra. These identities define Jordan trialgebras and post-Jordan algebras: Jordan analogues of the Lie trialgebras and post-Lie algebras introduced by Dotsenko et al., Pei et al., Vallette & Loday. We include an extensive review of analogous structures existing in the literature, and their interrelations, in order to identify the gaps filled by our two new varieties of algebras. We use computer algebra (linear algebra over finite fields, representation theory of symmetric groups), to verify in both cases that every polynomial identity of arity ≤ 6 is a consequence of those of arity ≤ 4. We conjecture that in both cases the next independent identities have arity 8, imitating the Glennie identities for Jordan algebras. We formulate our results as a commutative square of operad morphisms, which leads to the conjecture that the squares in a much more general class are also commutative.

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