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**TRACE IDEALS AND CENTERS OF ENDOMORPHISM RINGS
OF MODULES OVER COMMUTATIVE RINGS**

HAYDEE LINDO

ABSTRACT. Let R be a commutative Noetherian ring and M a finitely generated R -module. Under various hypotheses, it is proved that the center of $\text{End}_R(M)$ coincides with the endomorphism ring of the trace ideal of M . These results are exploited to establish results for balanced and rigid modules, and to settle certain cases of a conjecture of Huneke and Wiegand.

1. INTRODUCTION

Let R be a commutative ring and M a finitely generated R -module. The trace ideal of M , denoted $\tau_M(R)$, is the ideal $\sum \alpha(M)$ as α ranges over $M^* := \text{Hom}_R(M, R)$. We are interested in the connection between the properties of M and those of $\tau_M(R)$. For special classes of modules the corresponding trace ideals demonstrate remarkable properties. For example, a finitely generated ideal is the trace ideal of a projective module if and only if it is idempotent, and $\tau_M(R) = R$ if and only if every finitely generated R -module is a homomorphic image of a direct sum of copies of M ; see [9], [19] and Section 2.

This work was motivated by hints in the literature about the relationship between $\tau_M(R)$ and the center of $\text{End}_R(M)$. In [1], Auslander and Goldman show that when $\tau_M(R) = R$, each endomorphism in the center of $\text{End}_R(M)$ is given by multiplication by a unique ring element (see also [7, Exercise 95]). That is, $Z(\text{End}_R(M)) = R$.

Two of our central results clarify this connection, first in the case when M is reflexive and faithful and second in the case where $\tau_M(R)$ has positive grade; see Theorems 3.9 and 3.21. In both cases, $\text{End}_R(\tau_M(R))$ equals the center of the endomorphism ring of an associated module (when the rings are viewed as subrings of the total ring of quotients); see Corollaries 3.17 and 3.24.

Theorem. *Let R be a Noetherian ring and M a finitely generated R -module.*

- (i) *If M is reflexive and faithful, then there is a canonical isomorphism of R -algebras*

$$\text{End}_R(\tau_M(R)) \cong Z(\text{End}_R(M))$$

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