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Haydee Lindo

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TRACE IDEALS AND CENTERS OF ENDOMORPHISM RINGS OF MODULES OVER COMMUTATIVE RINGS

HAYDEE LINDO

ABSTRACT. Let R be a commutative Noetherian ring and M a finitely generated R-module. Under various hypotheses, it is proved that the center of $\mathrm{End}_R(M)$ coincides with the endomorphism ring of the trace ideal of M. These results are exploited to establish results for balanced and rigid modules, and to settle certain cases of a conjecture of Huneke and Wiegand.

1. INTRODUCTION

Let R be a commutative ring and M a finitely generated R-module. The trace ideal of M, denoted $\tau_M(R)$, is the ideal $\sum \alpha(M)$ as α ranges over $M^* := \mathrm{Hom}_R(M, R)$. We are interested in the connection between the properties of M and those of $\tau_M(R)$. For special classes of modules the corresponding trace ideals demonstrate remarkable properties. For example, a finitely generated ideal is the trace ideal of a projective module if and only if it is idempotent, and $\tau_M(R) = R$ if and only if every finitely generated R-module is a homomorphic image of a direct sum of copies of M; see [9], [19] and Section 2.

This work was motivated by hints in the literature about the relationship between $\tau_M(R)$ and the center of $\mathrm{End}_R(M)$. In [1], Auslander and Goldman show that when $\tau_M(R) = R$, each endomorphism in the center of $\mathrm{End}_R(M)$ is given by multiplication by a unique ring element (see also [7, Exercise 95]). That is, $\mathrm{Z}(\mathrm{End}_R(M)) = R$.

Two of our central results clarify this connection, first in the case when M is reflexive and faithful and second in the case where $\tau_M(R)$ has positive grade; see Theorems 3.9 and 3.21. In both cases, $\mathrm{End}_R(\tau_M(R))$ equals the center of the endomorphism ring of an associated module (when the rings are viewed as subrings of the total ring of quotients); see Corollaries 3.17 and 3.24.

Theorem. *Let R be a Noetherian ring and M a finitely generated R-module.*

(i) If M is reflexive and faithful, then there is a canonical isomorphism of R-algebras

$$\mathrm{End}_R(\tau_M(R)) \cong \mathrm{Z}(\mathrm{End}_R(M))$$

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