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The left and right dimensions of a skew field over the subfield of invariants

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Abstract

If H is a Hopf algebra and A an H-module algebra without nontrivial H-stable left or right ideals, then the subalgebra of H-invariant elements A^H is a skew field and A may be regarded as a vector space over A^H with respect to either left or right multiplications. It is proved in the paper that the left dimension of A over A^H is equal to the right dimension under the assumptions that A is semiprimary and $\dim H < \infty$. In the case when A is itself a skew field, this answers a question raised by J. Bergen, M. Cohen and D. Fischman.

Keywords: Hopf algebras, Hopf module algebras, skew fields, invariants
2010 MSC: 16T05

Introduction

Let A be a left H-module algebra where H is a finite dimensional Hopf algebra over a field k. In the case when A is a skew field, i.e., a division ring, the subring of H-invariant elements A^H is a skew field too, and both the left dimension $[A : A^H]_l$ and the right dimension $[A : A^H]_r$ of A over A^H are finite. In a 1990 paper Bergen, Cohen and Fischman asked whether the equality $[A : A^H]_l = [A : A^H]_r$ always holds [1, Question 2.4]. This question was motivated by the classical result that such an equality is indeed true in the case of group actions on skew fields [7]. For arbitrary extensions of skew fields the problem of Artin concerning equality of the left and right dimensions had been solved in the negative, even when both quantities are assumed to be finite [4], [10].

In the present paper we answer the question of Bergen, Cohen and Fischman. Moreover, the assumption that A is a skew field can be weakened. Recall that a ring is said to be *semilocal* if the factor ring by its Jacobson radical is semisimple artinian. A semilocal ring is *semiprimary* if the Jacobson radical is nilpotent.

Theorem 0.1. *Let H be a finite dimensional Hopf algebra over a field. Suppose that A is a semiprimary H-module algebra without nontrivial H-stable left (or right) ideals. Then A^H is a skew field and $[A : A^H]_l = [A : A^H]_r$.*

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