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Coherence for monoidal G-categories and braided G-crossed categories

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ABSTRACT

We prove a coherence theorem for actions of groups on monoidal categories. As an application we prove coherence for arbitrary braided G-crossed categories.

1. Introduction

Given a group G and a monoidal category $\mathcal{C}$, a *strict action* of G on $\mathcal{C}$ is a group morphisms from G to $\mathrm{Aut}_{\otimes}^{\mathrm{Strict}}(\mathcal{C})$ (the group of all strict monoidal automorphisms of $\mathcal{C}$). For almost all situations where symmetries of monoidal categories arise, strict actions are not sufficient, mainly because the natural notion of symmetry in monoidal category theory is not a strict monoidal automorphism. Rather, a symmetry in a monoidal category is a strong monoidal *auto-equivalence*. The categorical symmetries of a monoidal category form a monoidal category. Since every group G defines a discrete monoidal category $\overline{G}$, the appropriate definition of action is a monoidal functor from $\overline{G}$ to $\mathrm{End}_{\otimes}(\mathcal{C})$,

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where $\mathrm{End}_{\otimes}(\mathcal{C})$ is the monoidal category of strong monoidal endofunctors as objects and morphisms given by monoidal natural isomorphism. For simplicity, a monoidal category with a G-action will be called a monoidal G-category. If the action is strict we will say that the monoidal G-category is strict. Thus, a monoidal G-category is a monoidal category $\mathcal{C}$ with a family of monoidal auto-equivalences $\{(g_*, \psi^g) : \mathcal{C} \to \mathcal{C}\}_{g\in G}$ and a family of monoidal isomorphisms $\{\phi(g,h) : (gh)_* \to g_*h_*\}_{g,h\in G}$ satisfying certain coherence axioms (see Section 3.1).

The distinction between strict monoidal G-categories and general monoidal G-categories is analogous to the relation between strict monoidal categories and general monoidal categories. In monoidal category theory, the MacLane coherence theorem says that for two expressions S_1, S_2 obtained from $X_1 \otimes X_2 \otimes \cdots \otimes X_n$ by inserting $\mathbf{1}$'s and brackets, any pair of isomorphisma $\Phi : S_1 \to S_2$, composed of the associativity and unit constraints and their inverses, are equal. A similar presentation of coherence for arbitrary monoidal G-categories can be stated. However, an equivalent statement and convenient way of expressing the coherence theorem for monoidal G-categories is the following:

Theorem 1.1 *(Coherence for monoidal G-categories). Let G be a group. Every monoidal G-category is equivalent to a strict monoidal G-category.*

In essence, Theorem 1.1 says that in order to prove a general statement for monoidal G-categories, we may assume without loss of generality to assume that we are working with strict G-categories.

The main result of this paper is to prove Theorem 1.1. For this, given a group G and a monoidal G-category $\mathcal{C}$, we construct a strict monoidal G-category $\mathcal{C}(G)$ and a adjoint monoidal G-equivalence $\mathcal{F} : \mathcal{C} \to \mathcal{C}(G)$. In fact, we construct a strict left adjoint 2-functor to the forgetful 2-functor from the 2-category of strict monoidal G-categories to the 2-category of monoidal G-categories.

Braided G-crossed categories are interesting because they have applications to mathematical physics [2,6,8] and low-dimensional topology [12–14]. As an application of Theorem 1.1, we prove coherence theorems for general G-crossed categories and braided G-crossed categories. This coherence theorem generalizes the Müger's coherence theorem for braided G-crossed fusion categories over algebraically closed field of characteristic zero and G finite, [12, Appendix 5, Theorem 4.3]. Müger's coherence theorem is obtained as corollary of depth and difficult to prove characterization of braided G-crossed fusion categories. The inconveniences of [12, Appendix 5, Theorem 4.3] are that the construction of the strictification is not direct, G must be finite and the conditions on the braided G-crossed category is very restrictive. In [12] Michael Müger asked for a prove of the coherence in a more direct way, extending its domain of validity. Theorem 5.6 has no restriction on G or the underlying category $\mathcal{C}$ and the constructions of the strictification $\mathcal{C}(G)$ is very explicit. Thus, Theorem 5.6 answers Müger's question.

The paper is organized as follows. Section 2 contains preliminaries and notations. In Section 3, we define the 2-category of monoidal G-categories. Section 4 contains the proof