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Igusa–Todorov functions for radical square zero algebras ☆

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ABSTRACT

In this paper we study the behaviour of the Igusa–Todorov functions for radical square zero algebras. We show that the left and the right ϕ-dimensions coincide, in this case. Some general results are given, but we concentrate more in the radical square zero algebras. Our study is based on two notions of hearth and member of a quiver Q. We give some bounds for the ϕ and the ψ-dimensions and we describe the algebras for which the bound of ψ is obtained. We also exhibit modules for which the ϕ-dimension is realised.

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1. Introduction

One of the most important conjectures in the Representation Theory of Artin Algebras is the finitistic dimension, which states that the $\sup\{\mathrm{pd}(M) : M$ is a finitely generated module of finite projective dimension$\}$ is finite. As an attempt to prove the conjecture Igusa and Todorov defined in [6] two functions from the objects of $\mathrm{mod}A$ to the natural numbers, which generalises the notion of projective dimension, they are known, nowadays, as the Igusa–Todorov functions, ϕ and ψ. One of its nicest features is that they are finite for each module, and they allow us to define the ϕ-dimension and the ψ-dimension of an algebra. These are new homological measures of the module category and in fact it holds that $\mathrm{findim}(A) \leq \phi\dim(A) \leq \psi\dim(A) \leq \mathrm{gldim}(A)$ and they all coincide in the case of algebras with finite global dimension.

In [7] there are various relations of the ϕ-dimension with the bifunctors $\mathrm{Ext}(\cdot,\cdot)$ and $\mathrm{Tor}(\cdot,\cdot)$, they also prove that the finiteness of the this dimension is invariant for derived equivalence. Recently various works were dedicated to study and generalise the properties of these functions, see for instance [4,5,9], in particular in [9] the Igusa–Todorov functions were defined for the derived category of an Artin Algebra. The calculation of the values of these functions has not been done, up to now, for a large class of algebras. In this work we concentrate on the case of radical square algebras. Various notions related to this particular case are defined and worked out. We construct modules for which the ϕ-dimension is realised. We also give bounds for ϕ and ψ dimensions and discuss the cases where these bounds are obtained. We also give a complete characterisation of algebras with maximal ψ-dimension.

2. Preliminaries

We start fixing some notation, A will always denote an Artin algebra of type $A = \frac{\mathbb{K}Q}{I}$ where Q is a finite connected quiver, I is an admissible ideal, and $\mathbb{K}$ a field, these algebras are called elementary, [1]. All modules will be finitely generated right modules. The category of finitely generated A-modules will be denoted by modA. We will denote by J the ideal generated by the arrows in $\mathbb{K}Q$ and by $A_0 = \frac{\mathbb{K}Q}{J}$, which is the sum of all simple modules, up isomorphism. Given an A-module M we will denote its projective dimension by $\mathrm{pd}(M)$, and by $\Omega^n(M)$ and $\Omega^{-n}(M)$ the nth syzygy and nth cosyzygy of M with respect to a minimal projective resolution and a minimal injective coresolution, respectively. We recall that the global dimension of A which we will denote by $\mathrm{gldim}(A)$ is the supremum of the set of projective dimensions of the A-modules, which is a natural number or infinite. The finitistic dimension of A, denoted by $\mathrm{findim}(A)$ is the supremum of the set of projective dimensions of the A-modules with finite projective dimension.

We will divide the set of isomorphism classes of simple modules into three distinct sets which are the following:

- $\mathcal{S} = \{S_1, \ldots, S_n\}$ denotes a complete set of simple A-modules, up to isomorphism.