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Stratifying endomorphism algebras using exact categories[☆]

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We dedicate this paper to the memory of J.A. Green

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ABSTRACT

This paper constructs enlargements of Hecke algebras over $\mathbb{Z}[t, t^{-1}]$ to certain standardly stratified algebras. The latter are obtained as endomorphism algebras of modules with dual left cell module filtrations in the sense of Kazhdan–Lusztig. A novel feature of the proofs is the use of suitably chosen exact categories to avoid difficult Ext^1-vanishing conditions.

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1. Introduction

This paper is the second in a series aimed at proving versions of a conjecture made by the authors in 1996. The conjecture concerns the enlargement, in a framework involving Kazhdan–Lusztig cell theory, of those Hecke endomorphism algebras which occur naturally in the cross characteristic representation theory of finite groups of Lie type. See [6] for the original version of the conjecture, and [7] for a reformulation.

The [6] conjecture is set in the context of a Hecke algebra $\mathcal{H}$ for a finite Weyl group, using the dual left cell modules S_ω, $\omega \in \Omega$, in the sense of [12]. (Thus, each S_ω is a right $\mathcal{H}$-module.) The base ring (in [7]) is $\mathbb{Z}[t, t^{-1}]$, where t is an indeterminate. One of the conjecture's implications is that there is a faithful right $\mathcal{H}$-module $T^\dagger$, filtered by various S_ω, such that the modules $\Delta(\omega) := \mathrm{Hom}_{\mathcal{H}}(S_\omega, T^\dagger)$, with $\omega \in \Omega$, form a stratifying system (in the sense of [6]) for the endomorphism algebra $A^\dagger := \mathrm{End}_{\mathcal{H}}(T^\dagger)$. Using exact category methods, we are able to prove this statement. See Theorem 4.9 below.

A strength of the "stratifying system" construction is that it is well-behaved under base change, so that the resulting algebra $A^\dagger \otimes k$ inherits a stratification from that of $A^\dagger$ over any Noetherian commutative ring or field k in which t is specialized to an invertible element.

The endomorphism algebras $A^\dagger$ constructed here have other good properties. In particular, based changed versions $\widetilde{A}^\dagger, \widetilde{T}^\dagger$ can be shown to satisfy the particular "cyclotomic" local versions of the conjecture which were treated in [7, Theorem 5.6], using results of [9] on the module categories $\mathscr{O}$ for rational Cherednik algebras. The present paper raises the possibility that the [6] conjecture can be proved directly within the global framework of $\mathbb{Z}[t, t^{-1}]$-algebras and modules, perhaps with the present $A^\dagger$, or a close variation.

The authors began developing a general theory in [6] for constructing the required enlarged algebras, centered around a set of requirements contained in what we call the "stratification hypothesis." The most difficult condition to verify in this hypothesis is an Ext^1-vanishing requirement for some of the modules involved. The present paper takes a novel approach to this problem by building new exact categories containing the relevant modules, effectively making the Ext^1-groups involved smaller and better behaved. While there are Specht modules and analogues for all finite Weyl groups, there are no troublesome self-extensions, or extensions in the "wrong order," because of the exact structure we construct. As a result, many issues of "bad characteristic" do not arise.

The present paper also contains new results on exact category constructions. In particular, the main Lemma 3.1 gives a very general construction in an abstract setting. It very quickly leads to new exact module categories $(\mathscr{A}, \mathscr{E})$ for algebras B over Noetherian domains $\mathscr{K}$, when the K-algebra B_K obtained by base change from $\mathscr{K}$ to its quotient field K is semisimple. The underlying additive category $\mathscr{A}$ is the full subcategory of B-mod consisting of all modules which are finitely generated and torsion-free over $\mathscr{K}$. The "exact sequences" in $\mathscr{E}$ are required to be exact on certain filtrations; see