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An action of the Hecke monoid on rational modules for the Borel subgroup of a quantised general linear group

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We construct an action of the Hecke monoid on the category of rational modules for the quantum negative Borel subgroup of the quantum general linear group. We also show that this action restricts to the category of polynomial modules for this quantum subgroup and induces an action on the category of modules for the quantised Borel–Schur algebra $S^-_{\alpha,\beta}(n,r)$.

In memory of J.A. Green, with the highest admiration.

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1. Introduction

The idea of "*categorification*" originates from the joint work [3] of Crane and Frenkel, and the term was coined later in Crane's article [2]. Recently *categorification* became an intensively studied subject in several mathematical areas. A detailed account on this topic can be found in [12].

Given an action ρ of a monoid M on the Grothendieck group $\mathrm{Gr}(\mathcal{C})$ of a category $\mathcal{C}$, one can ask if it can be categorified, that is if there exists an action of M on $\mathcal{C}$ that induces ρ. Note that, to find such an action, one has: (i) to find a set of functors F_m, $m \in M$, whose action on $\mathcal{C}$ gives operators $\rho(m)$ on $\mathrm{Gr}(\mathcal{C})$; (ii) to show the existence of a coherent family of natural isomorphisms $\lambda_{m,m'} \colon F_m F_{m'} \to F_{mm'}$. Usually, (ii) is much more tricky than (i).

Let B_n be the negative Borel subgroup of the general linear group of degree n over an algebraically closed field. Denote by Gr the Grothendieck group of the category of finite dimensional polynomial B_n-modules. The tensor product of modules turns Gr into a ring. Let N be a finite dimensional polynomial B_n-module. Then we can consider the formal character ch_N of N in $\mathbb{Z}[x_1, \dots, x_n]$. In fact ch is a ring homomorphism from Gr to $\mathbb{Z}[x_1, \dots, x_n]$. In [5] Demazure showed how the characters of certain B_n-modules can be calculated by applying what is now called the *Demazure operators*, π_i, $1 \le i \le n-1$, to a monomial $x_1^{k_1} \dots x_n^{k_n}$. It turns out that the operators π_i, $1 \le i \le n$, define an action of the Hecke monoid, $\mathfrak{H}(\Sigma_\mathrm{n})$, on $\mathbb{Z}[x_1, \dots, x_n]$. Later Magyar in [11] generalised Demazure's character formula for the class of flag Weyl modules corresponding to percentage-avoiding shapes. While researching this class we were led to the idea that a categorification of the action of $\mathfrak{H}(\Sigma_\mathrm{n})$ on $\mathbb{Z}[x_1, \dots, x_n]$ can be useful to prove some conjectures stated in [14].

In this article we show that the Hecke monoid acts on the category of rational modules for the quantum negative Borel subgroup of the quantum general linear group. In fact, we construct what we call a *preaction* of $\mathfrak{H}(\Sigma_\mathrm{n})$ on this category. In [17] the second author proves that the category of actions of $\mathfrak{H}(\Sigma_\mathrm{n})$ on a category $\mathcal{C}$ is equivalent to the category of preactions of $\mathfrak{H}(\Sigma_\mathrm{n})$ on $\mathcal{C}$. Therefore, via this equivalence, from the constructed preaction one can obtain an action of $\mathfrak{H}(\Sigma_\mathrm{n})$ on the above referred category. It is then quite simple to get a preaction, and so an action, of $\mathfrak{H}(\Sigma_\mathrm{n})$ on the category of $S^-_{\alpha,\beta}(n,r)$-modules, where $S^-_{\alpha,\beta}(n,r)$ is the quantised (negative) Borel–Schur algebra. In a forthcoming paper we will show that the action of $\mathfrak{H}(\Sigma_\mathrm{n})$ on the category of rational modules for the quantum Borel subgroup induces an action of $\mathfrak{H}(\Sigma_\mathrm{n})$ on the corresponding derived category. This action will provide a categorification of the action of $\mathfrak{H}(\Sigma_\mathrm{n})$ on $\mathbb{Z}[x_1, \dots, x_n]$.

The paper is organised as follows. In Section 2 we introduce the notion of a preaction of the Hecke monoid $\mathfrak{H}(\Sigma_\mathrm{n})$ on a category $\mathcal{C}$. Section 3 contains some results, on cotensor product and induction for coalgebras, that are due to Takeuchi [15] and Donkin [7,8]. In Section 4 we study some subgroups of the quantum general linear group (or rather their coordinate Hopf algebras), namely quantum parabolic subgroups and quantum

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