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PRECOVERS AND ORTHOGONALITY IN THE STABLE MODULE CATEGORY

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ABSTRACT. We show that any module admits a presentation as the quotient of a Gorenstein projective module by a submodule which is itself right orthogonal, with respect to the standard Ext^1 pairing, to the class of Gorenstein projective modules of type FP_∞. For that purpose, we use the concept of orthogonality in the stable module category and examine the orthogonal pair which is induced therein by the class of completely finitary Gorenstein projective modules.

0. INTRODUCTION

Auslander has introduced the finitely generated modules of Gorenstein dimension zero over a commutative Noetherian ring in [2] and [3], as a generalization of the finitely generated projective modules. Since then, this concept has found many applications in commutative algebra and algebraic geometry. The generalization of this notion to any module over any ring by Enochs and Jenda in [14] lead to the definition of Gorenstein projective modules: These are the syzygy modules of the complete projective resolutions (a.k.a. totally acyclic complexes of projective modules), i.e. of the doubly infinite acyclic complexes of projective modules

$$\cdots \longrightarrow P_{n+1} \longrightarrow P_n \longrightarrow P_{n-1} \longrightarrow \cdots , \qquad (1)$$

which remain acyclic after applying the functor $\mathrm{Hom}_R(\_, P)$ for any projective module P. Modules of finite Gorenstein projective dimension can be defined in the standard way, by using resolutions by Gorenstein projective modules.

Avramov and Martsinkovsky studied in [1] the relative cohomology of finitely generated modules of finite Gorenstein projective dimension over a two-sided Noetherian ring. This has been taken up by Holm [17], who showed that the Gorenstein Ext functors (i.e. the relative derived functors of Hom, with respect to the class of Gorenstein projective modules) can be always defined on the subcategory of modules of finite Gorenstein projective dimension. The key property that modules M in that subcategory enjoy is the existence of *proper* Gorenstein projective resolutions: Such a module M admits an exact sequence of the form

$$\cdots \longrightarrow G_n \longrightarrow \cdots \longrightarrow G_1 \longrightarrow G_0 \longrightarrow M \longrightarrow 0,$$

where the modules G_i are Gorenstein projective for all $i \geq 0$ and the sequence remains exact after applying the functor $\mathrm{Hom}_R(G, \_)$ for any Gorenstein projective module G. It is through this latter condition that one can guarantee the independence of the Gorenstein Ext groups upon the choice of the particular resolution. The existence of proper Gorenstein projective resolutions in turn follows since any module M of finite Gorenstein projective dimension fits into an exact sequence

$$0 \longrightarrow K \longrightarrow G \xrightarrow{p} M \longrightarrow 0, \qquad (2)$$

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