Journal of Algebra 478 (2017) 261–270

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Invariable generation of Thompson groups ☆

Tsachik Gelander [a], Gili Golan [b,*], Kate Juschenko [c]

[a] *Weizmann Institute, Israel*
[b] *Vanderbilt University, United States*
[c] *Northwestern University, United States*

A R T I C L E   I N F O

Article history:
Received 25 November 2016
Available online 23 January 2017
Communicated by E.I. Khukhro

Keywords:
Invariable generation
Thompson groups

A B S T R A C T

A subset S of a group G *invariably generates* G if $G = \langle s^{g(s)} | s \in S\rangle$ for every choice of $g(s) \in G, s \in S$. We say that a group G is *invariably generated* if such S exists, or equivalently, if $S = G$ invariably generates G. In this paper, we study invariable generation of Thompson groups. We show that Thompson group F is invariably generated by a finite set, whereas Thompson groups T and V are not invariably generated.

1. Introduction

Recall that a subset S of a group G *invariably generates* G if $G = \langle s^{g(s)} | s \in S\rangle$ for every choice of $g(s) \in G, s \in S$. One says that a group G is *invariably generated*, or shortly IG, if such S exists, or equivalently if $S = G$ invariably generates G. The term "invariable

☆ The work of the first author was partially supported by ISF-Moked grant 2095/15. The work of the second author was partially supported by a Fulbright grant and a postdoctoral scholarship from Bar Ilan University. The work of the third author was partially supported by NSF CAREER grant DMS 1352173.

* Corresponding author.
E-mail addresses: tsachik.gelander@weizmann.ac.il (T. Gelander), gili.golan@vanderbilt.edu (G. Golan), kate.juschenko@gmail.com (K. Juschenko).

http://dx.doi.org/10.1016/j.jalgebra.2017.01.019

generation" was coined by Dixon [3] in his study of generation of Galois groups, where elements are given only up to conjugacy. Invariably generated groups were studied before by Wiegold [14,15] under different terminology: A group G is invariably generated if and only if no proper subgroup of G meets every conjugacy class. This is equivalent to saying that every transitive permutation representation of G on a non-singleton set admits a fixed-point-free element. Following [10], we say that G is *finitely invariably generated*, or shortly FIG, if there is a finite subset $S \subset G$ which invariably generates G.

A simple counting argument shows that every finite group is IG. Obviously, abelian groups are IG. More generally, J. Wiegold [14] showed that the class of IG groups is closed under extensions, hence contains all virtually solvable groups. Clearly, this class is also closed to quotients. On the other hand, the class of IG groups is not closed under direct unions; for instance the (locally finite) group of finitely supported permutations of $\mathbb{N}$ is clearly not IG, since every element fixes some point in $\mathbb{N}$. Moreover, Wiegold [15] gave an example of an IG group whose commutator subgroup is not IG, proving in particular that the class IG is not subgroup closed.

In [14], Wiegold proved that the free group $F = \langle a, b\rangle$ is not IG by producing a list L of conjugacy class representatives which are jointly independent (i.e., that freely generate a free group of infinite rank). To recall his construction, let $\{w_n\}$ be conjugacy class representatives which start and end with a non-zero power of b, then take $L = \{w_n^{a^n} : n \in \mathbb{N}\}$. T. Gelander proved in [4] that convergence groups, and in particular Gromov hyperbolic groups and relatively hyperbolic groups, are not IG, confirming a conjecture from [10]. Gelander and Meiri [5] established various examples of arithmetic groups possessing the Congruence Subgroup Property which are not IG, providing a negative answer to a question from [10].

The notion of finite invariant generation is more subtle. For example, it is still unknown weather every finitely generated solvable group is FIG. Kantor, Lubotzky and Shalev [10] proved that a finitely generated linear group is finitely invariably generated if and only if it is virtually solvable.

The main result of the paper is summarized in the following theorem:

Theorem 1. *Thompson's group F is finitely invariably generated. The Thompson groups T and V are not invariably generated.*

2. Thompson group F

2.1. F as a group of homeomorphisms

Recall that Thompson group F is the group of all piecewise linear homeomorphisms of the interval $[0, 1]$ with finitely many breakpoints where all breakpoints are finite dyadic and all slopes are integer powers of 2. The group F is generated by two functions x_0 and x_1 defined as follows [2]: