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Abstract

We consider an open problem posted by Sam and Snowden in 2014 in [12, Problem 11.5] when the category in question is $\mathrm{FI_G}$. We prove that, over a commutative coherent ring, every finitely presented FI-module is coherent. It turns out that many properties of finitely presented $\mathrm{FI_G}$-modules may be generalized from Noetherian rings to coherent rings.

Keywords: (Filtered) $\mathrm{FI_G}$-modules, Coherent rings.

2010 MSC: Primary 16G99, Secondary 18G10.

1. Introduction

Let G be a group. The category $\mathrm{FI_G}$, introduced in [13], is that whose objects are finite sets, and whose morphisms are pairs $(f, g) : S \to T$ such that $f : S \to T$ is an injection, and $g : S \to G$ is a map. Given two composable morphisms (f, g) and (f', g') in FI_G, we define $(f, g)(f', g') = (ff', h)$ where $h(x) = g'(x)g(f'(x))$. If $G = 1$ is the trivial group, then $\mathrm{FI_G}$ is equivalent to the category FI of finite sets and injections studied in [3].

Let R be a commutative ring.

By $\mathfrak{C}$ we denote the full subcategory of $\mathrm{FI_G}$ whose objects are $[n] := \{1, 2, \ldots, n\}$ for $n \in \mathbb{Z}_+$, the set of nonnegative integers. By convention, $[0] := \emptyset$. It is clear that $\mathfrak{C}$ is equivalent to $\mathrm{FI_G}$. We let $\underline{\mathfrak{C}}$ be the R-linearization of $\mathfrak{C}$.

A **$\mathfrak{C}$-module** V is a covariant functor V from $\mathfrak{C}$ to the category of R-modules. For $n \in \mathbb{Z}_+$, V_n denotes the value of V on n , and the symbol $M(n)$ stands for the free $\mathfrak{C}$-module $\underline{\mathfrak{C}}(n, -)$.

A $\mathfrak{C}$-module V is said to be **finitely generated** if there exists a surjective homomorphism $\bigoplus_{i \in I} M(n_i) \twoheadrightarrow V$ where I is some finite set and each $n_i \in \mathbb{Z}_+$. We say that $\mathfrak{C}$ is **locally Noetherian**, if every finitely

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