Simplicity of p-blocks of modular adjacency algebras of association schemes

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Abstract

A criterion is given for blocks of modular adjacency algebras of association schemes to be simple.

1 Introduction

In [4], the author proved the following theorem.

Theorem 1.1. *Let (X, S) be an association scheme, and let F be a field of characteristic p. Then the adjacency algebra FS is semisimple if and only if p does not divide the Frame number $\mathcal{F}(S)$.*

For the definition of the Frame number, see §2.4.

The notion of an association scheme generalizes the notion of a finite group, so that Theorem 1.1 is a straightforward generalization of Maschke's theorem on finite groups [7, III. Theorem 1.22].

Example 1.2. Let (X, S) be a thin association scheme defined by the symmetric group $\mathfrak{S}_3$ of degree 3. Then the Frame number of S is

$$\mathcal{F}(S) = 6^6 \times \frac{1 \times 1 \times 1 \times 1 \times 1 \times 1}{1 \times 1 \times 2^4} = 2^2 \times 3^6 = 2916.$$

So the adjacency algebra FS is semisimple if and only if $p \neq 2, 3$. But, in this case, the adjacency algebra is just the group algebra. So it is characterized by the group order $|\mathfrak{S}_3| = 6$ in Maschke's theorem.

For a finite group G and a field F of characteristic p, we know that

$$\begin{aligned} p \nmid |G| &\iff \text{the principal block of } FG \text{ is simple} \\ &\iff FG \text{ is semisimple.} \end{aligned}$$

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