Journal of Algebra 474 (2017) 134–165

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

# Characters of the norm-one units of local division algebras of prime degree

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ARTICLE INFO

*Article history:*
Received 16 January 2016
Available online 11 November 2016
Communicated by Dan Segal

*Keywords:*
Representation theory
Division algebras
Representation growth
$p$-adic analytic groups
Representation zeta function

ABSTRACT

We give an explicit construction of all complex continuous irreducible characters of the group $\mathrm{SL}_1(D)$, where $D$ is a division algebra of prime degree $\ell$ over a local field of odd residual characteristic different from $\ell$. For $\ell$ odd, we show that all such characters of $\mathrm{SL}_1(D)$ are induced from linear characters of compact-open subgroups of $\mathrm{SL}_1(D)$. We also compute an explicit formula for the representation zeta function of $\mathrm{SL}_1(D)$.

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http://dx.doi.org/10.1016/j.jalgebra.2016.11.004
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## 1. Introduction

Let $K$ be a local field with ring of integers $\mathfrak{o}$ of odd residual characteristic $p$, and let $D$ be a central division algebra over $K$. Let $\ell$ denote the degree of $D$, that is $\ell = (\dim_K(D))^{1/2}$. Let $L/K$ be an extension of degree $\ell$. The field $L$ embeds in $D$ as a maximal subfield and hence there exists an embedding of $D$ into the full matrix algebra $\mathbf{M}_\ell(L)$. The restriction of the determinant map to the image of $D$ in $\mathbf{M}_\ell(L)$ gives rise to the reduced norm map, denoted by Nrd. This definition is independent of the choice of extension $L/K$ and of the embedding of $L$ into $D$ (see [14]). The valuation map $val_K : K \to \mathbb{Z} \cup \{\infty\}$ extends uniquely to $D$ via the formula

$$val_D(x) = \frac{1}{\ell} val_K\left(\mathrm{Nrd}(x)\right) \quad (x \in D). \tag{1.a}$$

Let $SL_1(D)$ be the subgroup of elements of reduced norm 1 in $D^\times$. In this article we focus on the group $SL_1(D)$ and give an explicit construction of all complex continuous irreducible characters of $SL_1(D)$, under the assumption that $\ell$ is a prime number different from $p$. This construction enables us to determine the number of such irreducible characters of $SL_1(D)$ of any given dimension. In particular, we obtain a formula for the representation zeta function of $SL_1(D)$.

Given a topological group $\Gamma$ we write $\mathrm{Irr}(\Gamma)$ to denote the set of complex continuous irreducible characters of $\Gamma$. The representation zeta function of $\Gamma$ is defined by the Dirichlet generating function

$$\zeta_\Gamma(s) := \sum_{\chi \in \mathrm{Irr}(\Gamma)} \chi(1)^{-s} \quad (s \in \mathbb{C}). \tag{1.b}$$

The abscissa of convergence of $\zeta_\Gamma(s)$ is the infimum of all $\alpha \in \mathbb{R}$ such that the series in (1.b) converges on the complex right half-plane $\{s \in \mathbb{C} \mid \mathrm{Re}(s) > \alpha\}$.

In the case where $\Gamma$ is a compact $p$-adic analytic group (for $p > 2$), Jaikin-Zapirain has shown that the representation zeta function of $\Gamma$ is of the form

$$\zeta_\Gamma(s) = \sum_{i=1}^{r} n_i^{-s} f_i(p), \tag{1.c}$$

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